Steady Flow of Fluid through Equilateral Triangular Tube

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Keeping in view the importance of fluid mechanics in various applications in hydrology and chemical engineering, the aim of the present paper is to report an analysis of the steady flow of fluid through an equilateral triangular tube by taking into account the velocity slip and no velocity slip at the surface of the medium. The fluid motion and flux are obtained in both cases. The flow considered here is laminar and steady. Therefore, the velocity of fluid is a function of any distance taken from z-axis. A particular case *i.e.* the motion of frictionless fluid in a rotating vessel of the form of hyperbolic cylindrical section has been considered and important mathematical results and conclusions have been drawn.

INTRODUCTION

There is great importance of both steady and unsteady flow in porous and non-porous media. They play a major role in petroleum industry and hydrology concerning the migration of oil gas and water and in chemical engineering concerning the filtration process.[1–4] Some problems on specialised flow were studied by various workers but they were based on older theories of non-linear viscosity such as Rivilin and Erickson theory.[5] The problems were not fully explored in the light of Nolls simple fluid theory. Recently Hele show flow of a static viscous fluid. We have taken some problems of steady flow of fluid through an equilateral triangular tube based on Poiseuille's law.[6]

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Nomeneclature:

μ	absolute viscosity
u, v, w	velocity components
p_1, p_2	the values of the mean pressure
P	the mean pressure or pressure
r	the distance from origin
L	the length of pipe
A, B	integration constants
a	the length of sides of equilateral triangular pipes
$\vec{q}$	velocity of liquid normal to origin
p	the constant pressure gradient
a,b	semi-area of the hyperbolic cylindrical section (particular case)
q_0	mean velocity over the cross section.

Basic equation of the problem

We take the similar side of length a of equilateral triangular pipe. Here, z-axis is taken parallel to the line passing through the centre of gravity of the equilateral triangular tube.

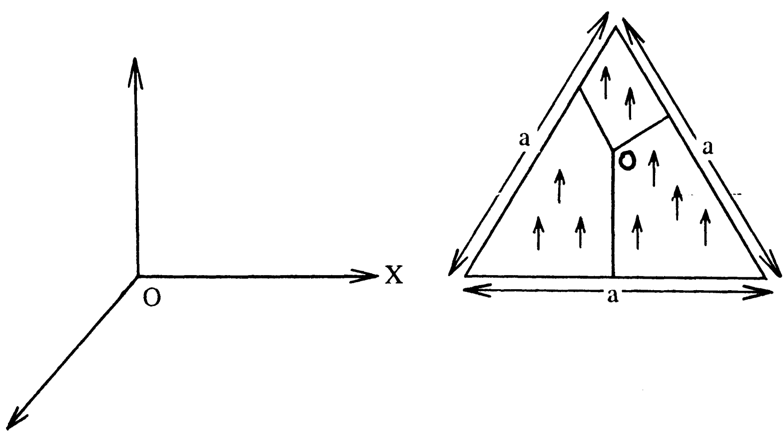

Fig. 1

The point O is taken as origin. It is obvious that the direction of velocity of fluid is parallel to z-axis everywhere. The velocity $\vec{q}$ is a function of distance r from the z-axis. Here $\frac{\partial q}{\partial z} = 0$ because the flow is perpendicular to x-y plane. The tangential stress across a plane perpendicular r is equal to $\mu \frac{\partial q}{\partial r}$; therefore due to tangential fractions on the two curved surfaces a retarding force is produced and this retarding force is balanced by the normal pressures on the plane end of the cylindrical shell.

Now we can write

$$-\frac{\partial}{\partial r}\left(\frac{\partial q}{\partial r} 2\pi r L\right) dr = (p_1 - p_2) 2\pi r\, dr$$

Hence

$$\frac{\partial}{\partial r}\left(r \frac{\partial q}{\partial r}\right) = -\frac{(p_1 - p_2) r}{\mu_L} \qquad (1.1)$$

Solution

Integrating (1.1) we get

$$q = \frac{(p_1 - p_2)}{4\mu L} r^2 + A \log r + B \qquad (2.1)$$

The boundary condition as such is that $q = 0$ at $r = 0$ and $q = 0$ at $r = \frac{a}{\sqrt{3}}$.

There is no slip at the wall of the pipe $\left(r = \frac{a}{\sqrt{3}}\right)$.

Using this boundary condition in (2.1) we get B = 0.

$$A = \frac{(p_1 - p_2)a^2}{12L\mu} \; \frac{1}{\log a - \frac{1}{2}\log 3}$$

Substituting the values of A and B in (2.1),

$$q = \frac{(p_1 - p_2)}{4\mu L}\left[a^2 \frac{1}{\log a^3 - 0.71568} - r^2\right] \tag{2.2}$$

Flux across any section:

$$\int_a^{a/\sqrt{3}} q\, 2\pi r\, dr = \frac{p_1 - p_2}{4\mu L} 2\pi \int_0^{q/\sqrt{3}} \left(\frac{a^2}{3\log a - 0.71568} - r^2\right) r\, dr$$

$$= \frac{2\pi(p_1 - p_2)}{4\mu L}\left[\frac{a^2}{3\log a - 0.71568}\cdot\frac{a^2}{6} - \frac{a^4}{36}\right]$$

$$= \frac{a^4\pi(p_1 - p_2)}{12\mu L}\left[\frac{1}{3\log a - 0.71568} - \frac{1}{6}\right] \tag{2.3}$$

It is assumed that flow takes place under pressure only. If we have extraneous force X acting parallel to length of the pipe, then

$$\text{flux} = \frac{(p_1 - p_2)\pi a^4}{12\mu L}\left[\frac{1}{3\log a - 0.71568} - \frac{1}{6} + \rho X\right] \tag{2.4}$$

Generally the component of gravity in the direction of the length of the tube is X. This result has the great importance of furnishing a conclusive fluid. There is no appreciable slipping of the fluid in contact with the wall. If we are to assume slipping coefficient β as

$$\beta = -\mu \frac{\frac{\partial q}{\partial r}}{q}$$

$$q = -\frac{\partial q}{\partial r} \quad \text{where } \lambda = \frac{\mu}{\beta}$$

This determines the constant A. The equation (2.1) shows that

$$q = \frac{p_1 - p_2}{4\mu L}\left[a^2 \frac{1}{3\log a - 0.71568} - r^2 + 2\lambda\frac{a}{\sqrt{3}}\right] \tag{2.5}$$

Corresponding value of flux is

$$\int_0^{a/\sqrt{3}} q\, 2\pi r dr = \frac{a^4\pi(p_1 - p_2)}{12\mu L}\left[\frac{1}{3\log a - 0.71568} - \frac{1}{6} + \frac{2\lambda}{a\sqrt{3}}\right] \tag{2.6}$$

For Poiseuille's law to hold true, we have

$$\frac{1}{3\log a - 0.71568} - \frac{1}{6} = 0$$

Hence, $$a = 173.4 \text{ cm}$$

Here we have found a new result in view of our problem. The Poiseuille's law holds true the side of equilateral triangular tube, a, is 173.4 cm. The equation (2.5) shows that the time of flux of a given volume of fluid varies directly as the length of the tube, inversely as the difference of the pressures at the two ends and also inversely as the fourth power of the length of the side of the equilateral triangular tube, a (= 173.4 cm).

Particular case

We are studying theoretically the motion of frictionless fluid in a rotating vessel of the form of hyperbolic cylindrical section.

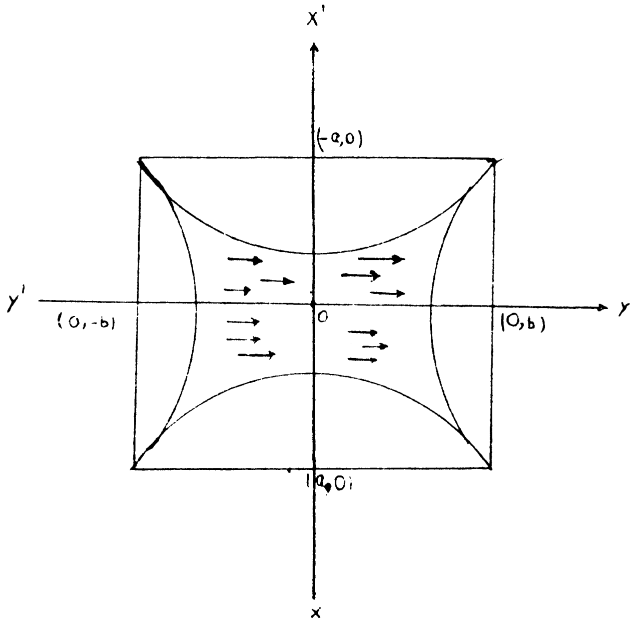

Fig.2. A cross section of hyperbolic cylinder

Formulation

We take the velocity components of fluid along x-axis and y-axis as zero respectively. It is assumed that q is a function of x and y only and u = 0 and v = 0. Here the z-axis coincides with the length of the hyperbolic cylinder.

Neglecting the inertial terms in the equation of motion of viscous fluid, the equation reduces to, in the absence of extraneous forces

$$\mu\nabla^2 q = \frac{\partial p}{\partial z} \tag{3.1}$$

We denote $\frac{\partial p}{\partial z} = P$ onwards.

Solution

From (3.1), we can write $\nabla^2 q = -\frac{p}{\mu}$

In the case of hyperbolic cylindrical section, we assume that

$$q_1 = A\left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right], \text{ for one sheet} \tag{3.2}$$

Also we assume that

$$q_2 = B\left[1 - \frac{x^2}{a^2} + \frac{y^2}{b^2}\right], \text{ for another sheet} \tag{3.3}$$

where a and b are semi axis of the hyperbolic cylinder. The equations (3.2) and (3.3) will satisfy

$$\nabla_1^2 q_1 = -\frac{p}{\mu} \tag{3.4}$$

and

$$\nabla_2^2 q_2 = \frac{-p}{\mu} \tag{3.5}$$

Equations (3.2) and (3.4) give

$$A = \frac{p}{2\mu}\frac{a^2b^2}{(a^2+b^2)} \tag{3.6}$$

Similarly from (3.3) and (3.5),

$$B = \frac{p}{2\mu}\frac{a^2b^2}{(b^2-a^2)} \tag{3.7}$$

Let, for whole hyperbolic cylinder, $q = q(q_1, q_2)$. In this case we assume that

$$q = A\left[1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}\right] + B\left[1 - \frac{x^2}{a^2} + \frac{y^2}{b^2}\right]$$

$$= (A+B) - (A+B)\frac{x^2}{a^2} - (A-B)\frac{y^2}{b^2} \tag{3.8}$$

where A and B are known from (3.6) and (3.7).

Here also

$$\nabla^2 q = \frac{-p}{\mu},$$

where p = constant pressure gradient

or

$$(A-B)\left[\frac{A+B}{A-B}\frac{1}{a^2} + \frac{1}{b^2}\right] = \frac{p}{2\mu} \tag{3.9}$$

Hence the discharge per second is equal to

$$\iint q\,dx\,dy = \int_0^a \int_0^{b^2} \frac{x^2}{a^2}\left\{(A+B) - (A-B)\frac{x^2}{a^2} - (A-B)\frac{x^2}{b^2}\right\} dx\,dy$$

$$= \frac{ab^2}{105}\left[\frac{p}{2\mu} a^2 b^2 \frac{(14-5b^2)}{(a^2+b^2)} + \frac{(14+5b^2)}{(b^2-a^2)}\right]$$

$$= \frac{p}{105\mu} \frac{a^3 b^6}{b^4 - a^4}(14+5a^2)$$

$$\therefore \quad \iint q\,dx\,dy = \frac{p}{105\mu} \frac{a^3 b^6}{b^4 - a^4}(14+5a^2)$$

Hence considerable variation may exist in the shape of the section without seriously affecting the discharge, provided the sectional area be unaltered. Here it is strictly considered that $b > a$

RESULTS AND DISCUSSION

Eqnation (2.2) gives the fluid motion of steady flow through equilateral triangular tube with no slip condition while equation (2.5) gives the fluid motion with the assumption of slip condition. Equation (2.3) gives flux across any section with no slip condition and equation (2.6) gives flux across section with slip condition.

In the steady case considerable variations exist in the shape of sections without seriously affecting the discharge provided the sectional area be unaltered.

Further, from (2.2) and (2.3) we find that $4q_0/a =$ rate of shear close to the wall of the channel where q_0 is the mean velocity over the cross section for values of q_0 exceeding certain limits depending on the relation between viscosity and length a; rectilinear flow becomes unstable and the motion becomes widely irregular, and in this position the motion of the fluid is turbulent. The case of flow through a pipe of circular section was the mode of the subject of a careful experimental study by Reynolds by means of filaments of coloured fluid introduced into the stream. So long as the mean velocity q_0 over the cross section falls below a certain limit depending on the radius of the pipe and the nature of the fluid, the flow is smooth and in accordance with Poisseuille's laws accidental disturbances are rapidly obliterated, and the region appears to be thoroughly stable. As q_0 is gradually increased beyond this limit, the flow becomes increasingly sensitive to small disturbances. But if care be taken to avoid these disturbances, smooth rectilinear character may for a while be preserved unntil at length a stage reached beyond which it is no longer possible. When the rectilinear regime definitely breaks down the motion becomes widely irregular and the tube appears to be filled.

Thus the curve (iv) illustrates with exaggerated amplitude the case of slightly disturbed stable steady motion parallel to an axis of permanent translation. The case of slightly disturbed unstable steady motion would be represented by a curve, continuous to (ii), on one side or the other, according to the nature of disturbance.

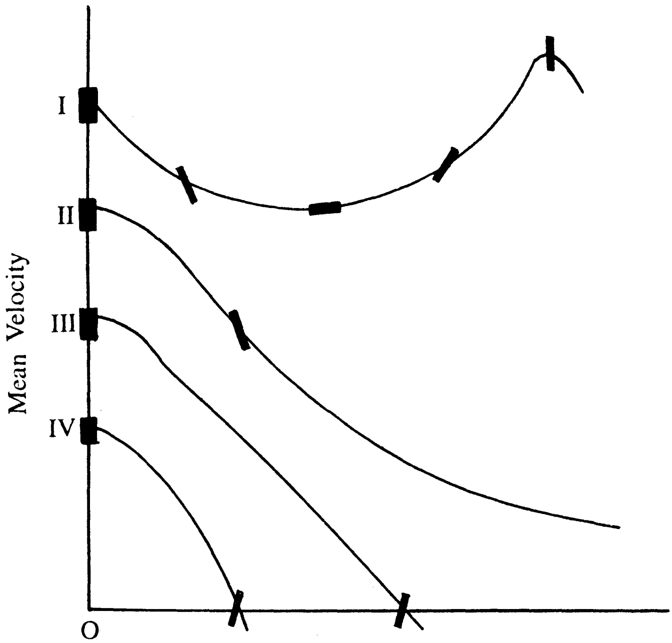

Fig. 3. Flow pattern of liquid.

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