

An Analogy-Based Model for Convective Heat Transfer Coefficient in Petroleum and Chemical Pipe

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Temperature is an important parameter for chemical reaction and the overall heat transfer coefficient is a key parameter for the temperature calculation in petroleum and chemical pipe. Driven by the radial temperature gradient, the heat flows out of the pipe continuously, but the engineers have long neglected its impact. In fact, the heat transfer in the boundary layer can never be ignored. By analogy between heat and momentum, a convective heat transfer coefficient model for petroleum and chemical pipe is established, based on which the overall heat transfer coefficient is calculated and analyzed. The results show that the convective heat transfer in the boundary layer has great influence on the overall heat transfer.

Key Words: Heat transfer coefficient, Chemical pipe, Pipe flow.

INTRODUCTION

In petroleum and chemical industry, the fluids usually contain many complex compounds, which have different physical and chemical property with different temperatures. During pipeline transportation, the temperature must be suitable or physicochemical changes would take place. In petroleum industry, the oil usually congeals and then blocks the pipeline under low temperature. In the chemical piping system, in order to avoid the unwelcome chemical reaction, the pipes also need to be insulated. If the heat insulation measures are insufficient, satisfactory effect cannot be obtained.

Overall heat transfer coefficient is a key parameter for pipe temperature simulation, because it determines the heat transfer amount between pipe and the environment. However, it cannot be calculated accurately, therefore, empirical value is often taken. The error of empirical value is evident, so if the accuracy of overall heat transfer coefficient can be improved, it would be of practical significance in the petroleum and chemical engineering.

Due to too many influencing factors, it is difficult to investigate the heat transfer of pipe flow in a theoretical manner. A flow and heat transfer model was constructed by Lu and Wang¹ to simulate the pipeline's shutdown with the environment temperature below zero. The phase change of the water in the soil, the solidification of the oil and the initial temperature were taken into consideration. Thermal analysis of the phase change in the horizontal insulated pipe was carried out by Bronfenbrener and Korin², who set up a quasistable state model. By using a software Comsol Multiphysics, Barletta *et al.*³ researched the heat transfer of an offshore berried pipe with the seabed temperature changing yearly.

The research works directly on the pipeline hydraulic and thermal calculation emphasize particularly on engineering issues and the information they conveyed is of great applicable value. Through analyzing the heat balance of a micro unit of the pipe, Guo⁴ developed a temperature distribution model in the insulation layer and in the pipe. Then he implemented the steady state and transient state simulation for the functioning of the pipeline. A gas transportation system composed of compressors and pipeline were evaluated with ant colony optimization (ACO) by Chebouba et al.⁵. Thuc⁶ have analyzed the heat and mass transfer of highly paraffin and highly congealing oil in a subsea oil pipeline. Hooker⁷ thought that the published literatures always neglected the compressibility of the fluid, but the actual temperature change was infected by both fluid friction and thermal expansion. Hooker investigated the fluid friction and thermal expansion coefficients and developed a steady state model for buried pipeline. Rawat⁸ has studied the heat conductivity coefficient of the soil and Fleyfel⁹ has evaluated the influence of the insulating layers and active heating.

At present the thermal models has not taken all the factors sufficiently. In the Leapienzon and Sukhov formulas, all the complicated factors are concealed in the overall heat transfer coefficient. In practice, the overall coefficient is often computed according the temperature measurements. The heat convection between the fluid and the pipe wall is affected by many influence factors, such as flow rate, physical properties of the fluid and pipe diameter. Therefore, if the conditions change the calculated overall heat transfer coefficient will no longer be correct. So this method can only be effective for one pipe and not for another.

The experiments are expected to give comparatively reliable result, but the parameters in engineering practice often exceed the test range. In petroleum industry, the radius of the oil pipe is usually very large and the radius of some newly built pipelines is even over one meter. Therefore it is necessary to develop a theoretical method to calculate the over heat transfer coefficient.

MATHEMATICAL MODEL

Under general velocity, the flow pattern of the pipe flow is usually in turbulent condition. The fluid can mix sufficiently in the main flow region, so there is no radial temperature gradient. If the temperature keeps uniform in the whole cross section, there will be no heat transfer in the radial direction. However, the flow in the laminar sublayer is laminar flow, in which the temperature changes violently. Under the temperature gradient in the boundary the heat transfer to or from the outside environment.

No matter how long the pipe is, the local condition determines the pipe fluid and the environment. So we must analyze the flow pattern and boundary condition. We can see that if the fluid has a uniform temperature in the radial direction, that is to say there is no temperature gradient, then there will be no heat exchange towards the outside environment. Because the pipe wall is in contact with the soil or the air, which have a lower temperature, the near wall fluid must have a lower temperature than the mainstream. In conclusion, there must be a transition layer, where the temperature drops from the mainstream temperature T to the near wall temperature T_w . Driven by the temperature in the transition layer, the heat flows towards the outside continuously.

The local flow pattern is shown in Fig. 1. Let it be supposed that the fluid in the mainstream has uniform temperature and velocity. In the temperature boundary layer the velocity and temperature come down to the near wall value. Driven by the temperature gradient in this area, heat flows to the pipe wall and then diffuses in the pipe wall and the soil by heat conduction. In this process, the convective heat transfer coefficient, the thermal conductivity of the pipe wall and the soil are crucial for calculation. The thermal conductivity of various soils can be determined by measurements, while the convection heat transfer coefficient is difficult to compute or measure.

Up to now, due to its complexity the knowledge of the turbulent flow is limited. Therefore, the convective heat transfer problem cannot be solved in theory and it can only be treated by analogy between heat and momentum approximately. Since the basic mechanism of both heat and momentum transfer is radial mixing of the fluid element, the convection heat transfer coefficient can be calculated from the flow parameters by analogy.



Fig. 1. Schematic diagram of the pipe flow structure

In the traditional analog technique the turbulent thermal diffusivity and momentum diffusivity are neglected, namely

 $\alpha_t = \varepsilon_t = 0$. For most gases $Pr \approx 1$, so the expression for the analogy of the heat and momentum can be simplified and the temperature distribution can be predicted easily. In this paper, we try to study the convection heat transfer and the overall heat transfer coefficient by analogy of the heat and momentum, without neglecting the turbulent thermal diffusivity and momentum diffusivity.

The shear stress in the boundary layer can be expressed as

$$\frac{\tau}{\rho} = (\nu + \varepsilon_{\rm M}) \frac{\partial \overline{\rm u}}{\partial \rm y} \tag{1}$$

Then the momentum equation is as follows:

$$\overline{u}\frac{\partial\overline{u}}{\partial x} + \overline{v}\frac{\partial\overline{u}}{\partial y} - \frac{\partial}{\partial y}\left[(v + \varepsilon_{M})\frac{\partial\overline{u}}{\partial y}\right] + \frac{1}{\rho}\frac{d\overline{P}}{dx} = 0$$
(2)

From (1)(2) we can get

$$\rho \overline{u} \frac{\partial \overline{u}}{\partial x} + \rho \overline{v} \frac{\partial \overline{u}}{\partial y} - \frac{\partial \tau}{\partial y} + \frac{d\overline{P}}{dx} = 0$$
(3)

In the boundary layer the value of \overline{u} is small, so the $\rho \overline{u} \frac{\partial \overline{u}}{\partial x}$ can be neglected. Under the assumption that the flow in the boundary is Couette flow and the radial velocity at the pipe

wall is zero, we can hold that $\overline{u} = \overline{u}(y)$, then

$$\frac{\mathrm{d}\tau}{\mathrm{d}y} - \frac{\mathrm{d}P}{\mathrm{d}x} = 0 \tag{4}$$

$$\frac{\tau}{\rho} = (\nu + \varepsilon_{\rm M}) \frac{d\overline{u}}{dy} \tag{5}$$

Integrate (4) from τ_o at the wall to τ where the distance is

$$\frac{\tau}{\tau_{o}} = 1 + \left(\frac{dP}{dx}\right) \frac{y}{\tau_{o}}$$
(6)

where $\tau_{\rm o} = \left(\frac{\mathrm{dP}}{\mathrm{dx}}\right) \frac{\mathrm{R}}{2}$.

y.

The heat transfer in the boundary layer is affected by molecular diffusion and turbulent, as follows:

$$q = (\alpha + \varepsilon_t) C_P \frac{dT}{dy}$$
(7)

In the analogy theory Pr usually is assumed to be 1 or restricted to a certain extent, then the influence of the turbulent diffusivity could be neglected. But the Pr number of the fluids makes a great difference. The Pr of gas is about 1.0, which is the assumed condition of the analogy theory. But the Pr number of oil can reach over one hundred. Although by neglecting the turbulent diffusivity the momentum equation can be solved satisfactorily, the small turbulent diffusivity can influence the heat transfer greatly under high Prandtl number. The small diffusivity near the pipe wall does not mean that its occurrence probability is small, but that the probability of the eddy is low.

The turbulent momentum and heat diffusivity can be related by the turbulent Pr number:

$$Pr_{t} = \frac{\varepsilon_{M}}{\varepsilon_{t}}$$
(8)

It is difficult to measure the Pr_t accurately and most of the research work focuses on the gas, the Pr number of which is about 1.0. Due to the great difference of Pr, the result of gas cannot be applied to oil directly. Especially, the results have shown that the Pr of the near-wall fluid is usually very high and much attention is paid to this region.

Kwon *et al.*¹⁰ has summarized the research results of Pr_t , from which we choose the classical expression:

$$Pr_{t} = 0.9 + \frac{182.4}{Pr \, Re^{0.888}} \tag{9}$$

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According to the mixing length theory, $\epsilon_{\rm M} = l^2 \left(\frac{{\rm d} u}{{\rm d} y} \right)$,

 $l = \kappa y$. For the hydraulic smooth pipe, $\kappa = 0.40$.

So ϵ_t can be described by ϵ_M and Pr_t and the heat flux can be formulated as:

$$q = C_{p}(\alpha + Pr_{t} \cdot \varepsilon_{M}) \frac{dT}{dy}$$
(10)

From (5) and (10) we can get:

$$\frac{\tau}{q} = \frac{(\mu + \varepsilon_M)}{(\alpha + Pr_t \,\varepsilon_M)C_p} \cdot \frac{\frac{du}{dy}}{\frac{dT}{dy}}$$
(11)

Substituting (6) into (11),

$$\frac{\left(\tau_{0} + \left(\frac{dP}{dx}\right)y\right)}{q} = \frac{\mu + \varepsilon_{M}}{\alpha + Pt_{t}\varepsilon_{M}}\frac{du}{dT}$$
(12)

In order to get the expression of the convective heat transfer coefficient, we need to integrate eqn. 12. In the temperature boundary layer the temperature increase from T_w at the pipe wall to T_{∞} at the frontier of the boundary layer. The lower

integration limit of u is 0 at the pipe wall and the upper limit needs to be obtained through analysis of the velocity distribution in the boundary layer.

The momentum boundary and the temperature boundary have reflected the diffusivity level of momentum and heat. Pr could represent the relationship of them. For oil whose Pr > 1:

$$\frac{\delta}{\delta_{\rm T}} \propto \Pr^{0.3} \tag{13}$$

It can be seen from (13) that the thickness of the temperature boundary layer is less than that of the momentum boundary layer.

The pipe flow of the hot oil pipeline is usually in the hydraulic smooth region and the corresponding thickness of the laminar sublayer is as follows:

$$\delta = \frac{32.8d}{\text{Re}\sqrt{\lambda}} \tag{14}$$

where $\lambda = \frac{0.3164}{\text{Re}^{0.25}}$.

Generally, the relationship between the temperature boundary layer and the momentum boundary layer is as listed as follows:

$$\frac{\delta_t}{\delta} = \Pr^{-1.2} \tag{15}$$

It is supposed that the temperature in the boundary layer shows linear distribution

$$\mathbf{y} = \mathbf{aT} + \mathbf{b} \tag{17}$$

According to the boundary conditions: $T = T_w$, y = 0 and $T = T_0$, $y = \delta_t$, the following parameters could be determined:

$$a = \frac{\delta_t}{T_0 - T_w}$$
 and $b = \frac{T_w \delta_t}{T_w - T_o}$.

With the linear distributed temperature, the heat flux in the boundary layer is the same. Integrate (12) in the boundary and then the convective heat transfer coefficient can be obtained:

$$h = \frac{\tau_{o} + \frac{1}{2}a\left(\frac{dP}{dx}\right)\overline{T} + b\frac{dP}{dx}}{\frac{\nu + \varepsilon_{M}}{\alpha + Pr_{t}}\varepsilon_{M}}u_{\delta_{t}}}$$
(18)

where $\overline{T} = \frac{(T_{\infty} + T_w)}{2}$ and u_0 is the velocity of the main flow region in the pipe.

Now the model of convective heat transfer coefficient in the boundary layer is established and the overall heat transfer coefficient can be expressed as:

$$K = \frac{1}{\frac{1}{h} + \sum \frac{\delta_i}{k_i} + \frac{1}{\alpha_2}}$$
(19)

where α_2 is the thermal conductivity of underground soil.

RESULTS AND DISCUSSION

It is commonly believed that the convective heat transfer coefficient can be neglected in the overall heat transfer coefficient model. However, according to present results, it cannot be neglected. In fact, all the heat exchanged with the environment must be first transferred to the boundary of the flow region in the form of convection and then to the soil in the form of conduction. So the convective heat transfer coefficient determines the total amount of the heat loss and cannot be neglected.

In the pipeline engineering, the convective heat exchange is usually neglected. But according to the present result, the error is big. The overall heat transfer coefficient is plotted in high and low viscosity range, respectively in Figs. 2 and 3. As shown in Fig. 2, the overall heat transfer coefficient neglecting the convective heat transfer coefficient is about 0.675 W/(m °C) and when taking the convective heat transfer coefficient into account, this value fall down to 0.1 W/(m °C).



Fig. 2. Plots of overall heat transfer coefficient *versus* pipe flow velocity at comparatively low viscosity



Fig. 3. Plots of overall heat transfer coefficient *versus* pipe flow velocity at comparatively high viscosity

As shown in Fig. 3, in comparatively high viscosity range the overall heat transfer coefficient also increases with viscosity. But with different viscosity the overall heat transfer coefficient behaves differently. Under 100 mm²/s the overall heat transfer coefficient keeps steady, while for 20 and 50 mm²/s it increases slowly. It indicates that we cannot use high velocity to reduce the heat dissipation and to avoid the coagulation of the oil. The high viscosity will induce high heat transfer coefficient and flow friction. And the energy consumption for increasing the velocity is considerable. So we should take measures to reduce the viscosity first.

In Fig. 3, the overall heat transfer coefficient nearly keeps constant with respect to viscosity and velocity when neglecting the convective heat transfer between the fluid and the pipe wall. But we also find that the higher the viscosity, the smaller error is. When the viscosity is 100 mm²/s, the error is only about 0.1 W/(m °C).

In Fig. 4 overall heat transfer coefficient is plotted *versus* viscosity and the influence of the convective heat transfer coefficient is obvious. When neglected, the convective heat transfer coefficient can cause large error. The error in low viscosity area can reach 0.6 W/(m °C). Comparing with the 0.05 W/(m °C) convective heat transfer coefficient this figure seems to be very big. In the high viscosity range, the error still is 0.1 W/(m °C).



Fig. 4. Plots of overall heat transfer coefficient *versus* viscosity at different pipe flow velocity

The influence of pipe radius and coating thickness to over all heat transfer coefficient can be reflected in this model. The relation of the overall heat transfer coefficient and the pipe radius is shown in Fig. 5, from which we can find that the overall heat transfer coefficient reduces with the pipe radius obviously. There will be great discrepancy if neglect the convective heat transfer. This figure is 0.7-0.25 W/(m °C) and may grow with radius.

The influence of the insulation thickness is also evaluated. From Fig. 6 we can find out that the overall heat transfer coefficient declines with the thickness of the insulation layer more quickly under smaller pipe radius. So the same thickness of insulation layer can produce better effect on smaller radius.

Conclusion

Under small radius, high viscosity and speed, the heat transfer is more severe. Under low viscosity and large radius, the influence of the convective heat transfer coefficient should



Fig. 5. Plots of overall heat transfer coefficient *versus* pipe radius at different flow velocity, for kinematic viscosity = 100 mm²/s



Fig. 6. Plots of overall heat transfer coefficient *versus* coating thickness at different pipe radius

not be neglected. In addition, they are prone to be affected by the surroundings. Consequently, it is difficult to calculate the overall heat transfer coefficient. In engineering practice, the overall heat transfer coefficient is usually calculated according to the pipeline measuring data. The model this paper put forward is based on reasonable fluid and heat transfer theories and it can reflect the influence of some fluid factors.

ε

Nomenclature

- Pr Prandtl number
- q Heat flux (w/s^2)
- ρ Density (kg/m³)
- α Thermal diffusivity (m²/s)
- u Axial velocity (m/s)
- y Radial coordinate (m)
- t Temperature (K)
- H_x Water head (m)

- Turbulent thermal
- $\begin{array}{l} \text{diffusivity (m^2/s)} \\ \delta_i \end{array} \\ \text{Thickness of pipeline} \end{array}$
- coatings (m)
- C_p Heat capacity at constant pressure J/(kg K)
- Kinematic viscosity (mm²/s)
- ϵ_{M} Dddy momentum diffusivity (m²/s)
- τ_{o} Shearing stress at the wall (N/m^2)
- Pr, Turbulent Prandtl number
- H_d Waterhead at the start
 - point (m)

Р	Pressure (Pa)	δ	Thickness of laminar sub layer (m)
R	Radius (m)	u _o	Velocity of the main stream (m/s)
x	Axial coordinate (m)	δ_t	Thickness of temperature boundary layer (m)
Re	Reynolds number	T_{w}	Temperature at the inner wall (K)
κ	Dimensionless number	T _o	Temperature of the main stream (K)
d	Inner diameter (m)	L _x	Distance from the start point (K)
μ	Dynamic viscosity (P·s)	ΔZ_x	Elevation difference from the start point (m)
λ	Hydraulic friction coefficient	g	Gravitational acceleration (m/s^2)
l	Prandtl mixing length (m)	K	Overall heat transfer coefficient (W/m ² K)
ν	Radial velocity (m/s)	h	Convective heat transfer coefficient (W/m ² K)
P _x	Dynamic pressure at x (Pa)	k _i	Thermal conductivity of pipeline coating (W/m K)
τ	Shearing stress (N/m ²)	α_2	Pipeline-soil heat transfer coefficient (W/m ² K)

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