



Equivalent Moduli of Carcass Layer in Flexible Pipes†

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Flexible pipes, as a typical composite structure, are generally applied to the ocean oil engineering. The Carcass layer of flexible pipes, with its particular structural formation, demonstrates orthotropic properties and the moduli of the Carcass layer are not the material moduli. Just the moduli of the Carcass layer are calculated, the whole structure will be analyzed. In the present paper, the equivalent modulus equations of the Carcass layer are deduced with the use of the ideology that under the same loading, the similar material and the original material have the same displacement. This analytical method of equivalent moduli shows good capacity in the prediction of the behaviour of the flexible pipes, which provides practical and technical support for the application of unbonded flexible pipes.

Keywords: Flexible pipe, Carcass layer, Orthotropic, Equivalent modulus.

INTRODUCTION

Flexible pipes, which comprise of several different layers, are slender marine composite structures that are widely used in offshore applications. From the most inner layer to the outer layer, a Carcass, an internal pressure sheath made of polymeric material, an interlocked pressure armour layer, an anti-wear layer, two tensile armour layers and an outer sheath with each layer having a particular function have been arranged sequentially¹. All layers are free to slide with respect to each other. The outer plastic sheath layer protects the riser from surrounding seawater intrusion, external damages during handling and corrosion. The inner plastic sheath layer ensures internal fluid integrity and is made from polymer.

Current methodologies concerning flexible pipe are generally divided into two categories *i.e.*, analytical formulations and finite element models. The analytical models share many of the following simplifying assumptions, which significantly limit the range of applicability of the results: displacements and strains are small²⁻⁴; some coupling terms in total stiffness matrix are neglected⁴; the conventional elastic thin-walled theory can be assumed valid^{3,5}; tendons are restricted from rotating about their local helical axis; strains and/or stresses across the layer thickness are constant; thicknesses of layers remain constant during deformation³; plane sections remain plane⁴; ovalisation effects are neglected^{4,6}; contact and/or frictional effects are ignored⁷⁻¹¹; no slip occurs between layers¹²;

tendons are constrained to slide only along their own axis; tendons respond only axially, (bending and torsional stiffness neglected); the interlayer contact pressure is constant; the contribution of the plastic sheaths to the strength and stiffness of the pipe can be neglected; layers remain constantly in contact (no bird-caging effect); the radial deformation is the same for all layers^{3,6,13}; initial manufacturing residual stresses can be ignored¹⁰. Therefore all available analytical methods are restricted because of the complexity of modeling layers.

Two generic types exist for flexible pipes *i.e.*, unbonded flexible pipe without adhesive agents between the layers and bonded flexible pipe with the reinforcing bonded to an elastomeric matrix. A flexible pipe is comprised of several distinct layers. This modular assembly allows each layer to be made fit-for-purpose and independently adjusted to best meet a specific field development requirement. In these types of pipes, although the layers are independent, they are designed to interact with one another. The cross-section of a flexible pipe is a combined construction of spirally wound steel layers and thermoplastic materials. The main components are leak-proof thermoplastic barriers and corrosion-resistant steel layers such as Carcass and pressure armour layers. The helically wound steel wires at the helical armour layer give a high-pressure resistance and excellent bending characteristics to the whole system of assembly, which results in the flexibility and a superior dynamic behaviour.

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Flexible pipes are structures used under water and are therefore subjected to an external hydrostatic pressure especially in the installation phase. The internal interlocked helical layer, known as the Carcass, is used primarily to take up the external pressure. A pipe without the Carcass layer can carry only a minimal amount of external pressure, since both the plastic tubes and the helical tendons have very low stiffness in the radial direction. Even without any hydrostatic pressure applied, the axisymmetric loading would cause an unrealistically large stretch, which occurs due to low stiffness of the pipe (without the carcass layer) in the radial direction.

The structure of the Carcass layer causes the stiffness in axial direction to be totally different comparing to the circumferential one, so that an orthotropic model is a reasonable choice. This layer is made up of an interlocked profiled strip in an almost circumferential lay. It is wound in a helix with a short pitch with a lay angle close to 90° and with a gap between turns. The Carcass layer is shown in Fig. 1.



Fig. 1. Carcass layer profile

Carcass layer is an orthotropic layer and not in regular shape, as a result, the equivalent modulus is not the material modulus. The equivalent moduli E_R , E_T , E_Z and G_R , G_T , G_Z are analyzed in the following part. The subscript R, T and Z are the directions of the pipe which have been shown in Fig. 2.

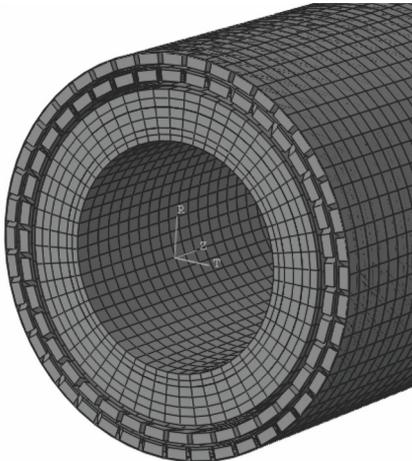


Fig. 2. Coordinate of the flexible pipe

METHOD

Equivalent moduli of the T direction: Here the Carcass layer profile is simplified as shown in the following Fig. 3.

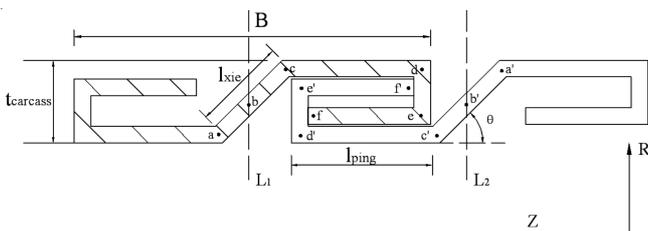


Fig. 3. Simplified Carcass profile

According to the ideological stiffness equivalent of Jose *et al.*¹⁴, the elastic modulus E_T , shear modulus G_T and the equivalent thickness t_{equ} of the T direction can be obtained.

Extension stiffness of the T direction is

$$E_{mat} A_{strip} = E_t t_{equ} (B - I_{ping}) \tag{1}$$

Bending stiffness of the T direction is

$$E_{mat} I_{bend} = E_t \frac{t_{equ}^3}{12} (B - I_{ping}) \tag{2}$$

Twist stiffness of the T direction is

$$G_{mat} I_{tor} = G_T \frac{t_{equ}^3}{3} (B - I_{ping}) \tag{3}$$

where E_{mat} and G_{mat} are the material modulus. Here, the steel is adopted the material AISI304 with the elastic modulus 190 GPa. So $E_{mat} = 190$ GPa. According to $G_{mat} = G_{mat}/2(1+\nu_{mat})$, G_{mat} can be achieved. A_{strip} , I_{bend} and I_{tor} can be obtained from the parameters of the Carcass layer.

According to the eqns. 1-3, can be obtained

$$t_{equ} = \sqrt{\frac{12I_{bend}}{A_{strip}}} \tag{4}$$

$$E_t = \frac{E_{mat} A_{strip}}{(B - I_{ping}) t_{equ}} \tag{5}$$

$$G_T = \frac{E_t G_{mat} I_{tor}}{4E_{mat} I_{bend}} \tag{6}$$

Equivalent moduli of Z direction

Modulus elasticity E_Z of Z direction: For cellular material, Fu and Yin¹⁵ considered the similar material and the original material have the same displacement under the same loading. The Carcass layer is similar to the cellular material and therefore, the same ideological can be used¹⁶. The equivalent structure is shown in Fig. 4. According to stress-strain relation, $\sigma = E\epsilon$, it can be obtained as

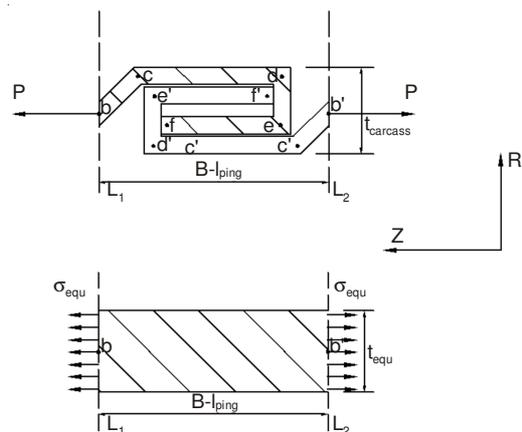


Fig. 4. Equivalent structure

$$E_Z = \frac{\sigma_{equ}}{\epsilon_Z} = \frac{P}{\frac{\delta_b}{B - I_{ping}}} \tag{7}$$

where, $\delta_b = 2\delta_{xie} + \delta_{ping}$ (8)
 δ_{xie} : the displacement of middle point b to point c. δ_{ping} : the horizontal distance of point c to point c'.

Solution of δ_{xie} : Forces in the rod are shown in Fig. 5. According to force equilibrium principle, can be obtained,

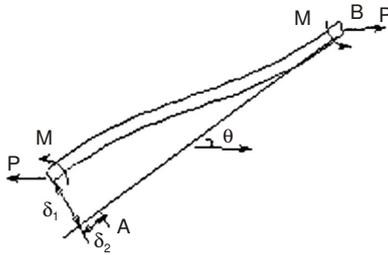


Fig. 5. Forces in the rod

$$M = \frac{1}{2} Pt_{carcass} \quad (9)$$

The displacement δ_1 of the middle point b of inclined rod ac is the sum of force P and moment M.

The displacement of the structure under the force P is

$$\delta = \frac{Pl^3}{3EI} \left[\frac{x^2}{l^2} \left(\frac{3}{2} - \frac{x}{l} \right) \right] \quad (10)$$

The displacement of the structure under the moment M is

$$\delta = \frac{Mx^2}{2EI} \quad (11)$$

So the total displacement of the middle point ($x = l/2$) is

$$\delta_1 = \frac{P \sin \theta l_{xie}^3}{24E_{mat} I} = \frac{P \sin \theta l_{xie}^3}{2E_{mat} t_{strip}^3} \quad (12)$$

where $I = bt^3_{strip}/12$, $b = 1$

Similarly, the displacement δ_2 along the rod is

$$\delta_2 = \frac{P \cos \theta}{E_{mat} t_{strip} b} \frac{l_{xie}}{2} = \frac{P \cos \theta l_{xie}}{2E_{mat} t_{strip}} \quad (13)$$

Substitute eqns. 12 and 13 into $\delta_{xie} = \delta_1 \sin \theta + \delta_2 \cos \theta$, can be obtained,

$$\delta_{xie} = \frac{Pl_{xie}}{2E_{mat} t_{strip}} \left(\frac{l_{xie}^2}{t_{strip}^2} \sin^2 \theta + \cos^2 \theta \right) \quad (14)$$

Solution of δ_{ping} : Here, two rods are under the compression force and the other two under the tensile force. According to force equilibrium, can be achieved as

$$\delta_{ping} = 2 \frac{Pl_{ping}}{E_{mat} t_{strip} b} + \frac{P}{2} \frac{l_{ping}}{E_{mat} t_{strip} b} \quad (15)$$

where, $b = 1$, then can be obtained,

$$\delta_{ping} = \frac{5Pl_{ping}}{2E_{mat} t_{strip}} \quad (16)$$

Equivalent modulus elasticity E_Z can be obtained by substituting eqns. 14 and 16 into eqn. 8.

Shear modulus G_Z : Here, G_Z adopts the same method with E_Z , the structure can be equivalent as the following form of Fig. 6. G_Z is

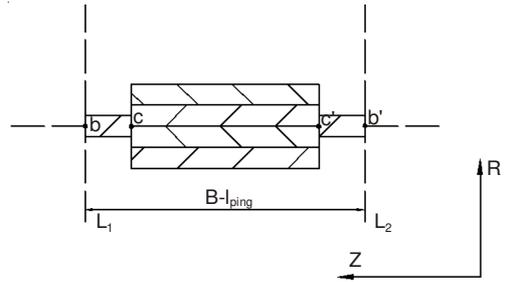


Fig. 6. Equivalent structure

$$G_Z = \frac{M}{\frac{\Phi_{niu}}{B - l_{ping}} \cdot I_{pequ}} \quad (17)$$

where $I_{pequ} = \frac{\pi(D_{equ}^4 - d_{equ}^4)}{32}$, $D_{equ} = D_{mid} + t_{equ}$, $d_{equ} = D_{mid} - t_{equ}$ and D_{mid} is the midline plane diameter of the Carcass layer. Φ_{niu} , combined by ϕ_1 and ϕ_2 , is the torsion angle between line L_1 and L_2 .

$$\Phi_{niu} = \phi_1 + \phi_2 \quad (18)$$

$$\phi_1 = \int_0^{l_{carcass}} \frac{M}{G_{mat} I_{pbc}} dZ + \int_0^{l_{carcass}} \frac{M}{G_{mat} I_{pb'c'}} dZ \quad (19)$$

$$\phi_2 = \int_0^{l_{ping}} \frac{M}{G_{mat} I_{pcc'}} dZ \quad (20)$$

$$I_{pbc} = I_{pb'c'} = I_{pstrip} = \frac{\pi}{32} (D_{strip}^4 - d_{strip}^4) \quad (21)$$

where $D_{strip} = D_{mid} + t_{strip}$, $d_{strip} = D_{mid} - t_{strip}$.

The thickness of cc' is four times of bc + b'c'. Thus,

$$I_{pc'c'} = 4I_{pstrip}$$

Substituting eqns. 18-21 into eqn. 17, it can be obtained as

$$G_Z = \frac{B - l_{ping}}{\frac{t_{carcass}}{\tan \theta} + \frac{l_{ping}}{4}} \frac{G_{mat} I_{pstrip}}{I_{pequ}} \quad (22)$$

Equivalent moduli of R direction: Elastic modulus E_R of radial direction is different under compression and tension. Since normally the Carcass layer will resist compression, in this scenario, we can only consider the equivalent elastic modulus E_R . Rod de and d'e' resist the load and the other stuff will not work. Then, E_R can be obtained as

$$E_R = \frac{\sigma_{equ}}{\epsilon_R} = \frac{F}{\frac{B - l_{ping}}{2E_{mat} t_{strip}}} = \frac{2E_{mat} t_{strip}}{B - l_{ping}} \quad (23)$$

For shear modulus, the method of shear strain equivalent will be adopted by

$$\gamma = \frac{\tau}{G_{mat}} = \frac{T}{AG_{mat}} = \frac{T}{2t_{strip} bG_{mat}} \quad (24)$$

$$\gamma = \frac{\tau'}{G_R} = \frac{T}{A'G_R} = \frac{T}{(B - I_{\text{ping}})bG_R} \quad (25)$$

As $\gamma = \gamma'$, it can be obtained as

$$G_R = \frac{2t_{\text{strip}}G_{\text{mat}}}{B - I_{\text{ping}}} \quad (26)$$

Consequently, the equivalent moduli E_R , E_T , E_z and G_R , G_T , G_z are calculated. With the moduli obtained, the mechanical function of the Carcass layer can be analyzed and the whole flexible pipe can be investigated.

Conclusion

Because the Carcass layer possesses the orthotropic properties and the irregular cross section profile, the equivalent elastic moduli are needed to be taken into account. The influence of deformation on structure stiffness has been developed in the analytical model. Consequently, the equivalent elastic moduli have been obtained, which resolves the problem concerning the calculation of equivalent moduli. In this method, the concrete form of the Carcass layer has taken into account for the first time. This analytical method of equivalent moduli shows good capacity in the prediction of the behaviours of the flexible pipes, which might provide practical and technical support for the application of unbonded flexible pipes.

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