



Simulation of Fracturing Process for Heterogeneous Material Using FLAC3D†

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AJC-15763

A novel numerical model for simulating the fracturing process of heterogeneous materials has been developed and implemented into the FLAC3D. A constitutive model that captures an essential component of brittle material failure *i.e.*, cohesion weakening and frictional strengthening is improved by taking into account that the elastic modulus of an element may degrade when the element fails. Furthermore, a new energy index, the local energy release rate (LERR), is proposed that simulates the energy released during a failure process through tracking the peak and trough values of the elastic strain energy intensity. The numerical model has been applied to simulate the fracturing processes of a heterogeneous material, such as rocks, under a triaxial loading condition are simulated by the proposed numerical model. The results demonstrate that the proposed model is a powerful approach for studying the macroscopic fracturing process of a heterogeneous material.

Keywords: Heterogeneous material, Fracturing process, Energy release, Numerical model.

INTRODUCTION

For mining and civil engineering as a whole, ground control is critical for constructing underground openings: *i.e.*, controlling the rock failure process that leads to a rock displacement surrounding the excavations. In many of the other engineering domains, a structure's behaviour under load can be analytically predicted with great certainty. However, rock engineering still has to rely on the properties and loading conditions that are encountered around underground openings. In deep level mining, where stresses often exceed the strength of the rock mass, an additional uncertainty is introduced by the behaviour of the rock during and after failure¹.

While the rocks and rocklike materials' behaviour and the presence of cracks or microcracks in rocks or rocklike materials are both influenced by various factors, a key influencer is heterogeneity. As is well known, a rock or rocklike material is heterogeneous with a composite or heterogeneous microstructure or texture; they also have preexisting or stress-induced defects. These features all indicate that microstructures are essentially heterogeneous and random. Consequently, heterogeneity should be taken into consideration by the numerical methods used for studying the fracture behaviour of rocks or rocklike materials². Therefore, different techniques have

been used to implement heterogeneity, including: Randomly assigning different properties, possibly following some kind of distribution, to the elements in all kinds of element network models³⁻⁶, using a mesh consisting of a random geometry but equal properties for the elements⁷, generating a microstructure and projecting this onto a regular element network and assigning different properties to the elements depending on their position⁸ and finally, using a combination of random geometry and a generated grain structure⁹. In this paper, a novel numerical model for simulating the fracturing process for a heterogeneous material is developed.

Constitutive model: The numerical simulation result of a heterogeneous material's failure process is strongly influenced by the constitutive model. Hajiabdolmajid *et al.*¹⁰ proposed a mobilization of strength constitutive model, the cohesion weakening and frictional strengthening (CWFS), which successfully modeled the brittle failure of rock. It considers the cohesion c and the friction angle ϕ of rock to be a function of equivalent plastic strain:

$$\tau = c(\epsilon^p) + \sigma_n(\epsilon^p) \tan \phi \quad (1)$$

where, τ is the shear strength of rock, σ_n is normal stress at failure face of rock and ϵ^p is equivalent plastic strain and can be expressed as:

†Presented at 2014 Global Conference on Polymer and Composite Materials (PCM2014) held on 27-29 May 2014, Ningbo, P.R. China

$$\varepsilon^p = \int \sqrt{\frac{2}{3} (d\varepsilon_1^p d\varepsilon_1^p + d\varepsilon_2^p d\varepsilon_2^p + d\varepsilon_3^p d\varepsilon_3^p)} dt \quad (2)$$

where $d\varepsilon_1^p$, $d\varepsilon_2^p$ and $d\varepsilon_3^p$ are increments of principal plastic strain. Important parameters of the CWFS model, including the initial cohesion, residual cohesion, maximum friction, c , φ , ε_c^p and ε_f^p , are shown in Fig. 1.

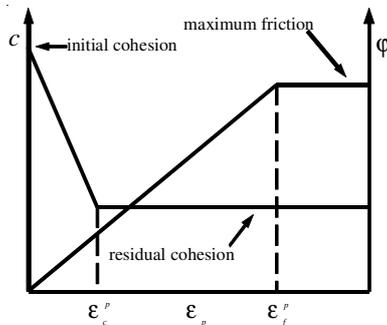


Fig. 1. Relationship curve of cohesion strength and friction strength vs. equivalent plastic strain

As observed through experiments¹¹, a descending elastic modulus and a descending load bearing capacity after a certain stage are the defining aspects of rock fragmentation under mechanical loading. However, the elastic modulus degradation has been neglected in the CWFS model. Therefore, we propose an improved CWFS model. According to the elastic damage mechanics, the element's elastic may degrade as:

$$E = d \times E_0 \quad (2)$$

where E and E_0 are the elastic modulus of the failure and undamaged material, respectively; d represents the degradation parameter.

Heterogeneous material model: The Weibull statistical distribution is used to characterize material heterogeneity¹². The two-parameter Weibull distribution can be expressed as:

$$P(\sigma) = \begin{cases} \frac{m x^{m-1}}{x_0^m} \exp\left[-\left(\frac{x}{x_0}\right)^m\right], & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (3)$$

where m is the shape parameter describing the scatter of x and x_0 is the scale parameter. In the numerical simulation, a heterogeneous material model is proposed that will characterize the heterogeneity of rock as the following: (1) the homogeneous index m , (2) the mean value of constitutive model parameters of elements and (3) $x_0 = 1$.

Energy index for brittle failure intensity assessment:

In this paper, a new index, the LERR (or local energy release rate), is proposed in order to quantitatively analyze failure intensity, based upon the understanding that an energy release is an intrinsic characteristic of brittle failure phenomena. It records the energy difference of the energy stored at the rock mass before and after the brittle failure: *i.e.*, the local energy release rate of the elements. However, it ignores the small energy release of the elements generated due to the non-brittle failure. An element's energy release rate generating the brittle failure is multiplicative with the element's volume in order to

obtain an elastic energy release of the element. The sum of the elastic energy release (ERE) from all the elements is the total elastic energy release of the brittle materials that occurred at the current loading step. The formula can be written by:

$$\text{LERR}_i = U_{i\max} - U_{i\min} \quad (4)$$

$$\text{ERE} = \sum_{i=1}^n \text{LERR}_i * V_i \quad (5)$$

where $U_{i\max}$ and $U_{i\min}$ are the elastic strain energy intensity values before and after the brittle failure occurrence at the i th element, respectively. V_i is the volume of the i th element.

$$U_{i\max} = \frac{[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)]}{2E} \quad (6)$$

$$U_{i\min} = \frac{[\sigma_1'^2 + \sigma_2'^2 + \sigma_3'^2 - 2\nu(\sigma_1'\sigma_2' + \sigma_2'\sigma_3' + \sigma_3'\sigma_1')]}{2E} \quad (7)$$

where, σ_1 , σ_2 and σ_3 are the three principal stresses that correspond to peak strain energy of the element. σ_1' , σ_2' , σ_3' are the three principal stresses that correspond to the element's lowest strain energy, ν is Poisson's ratio and E is Young's modulus.

Basic procedure: Generally, it is known that a material's fracture results from the initiation, growth and coalescence of microcracks. A numerical simulation must reflect this fracture process of a material subjected to loading. The basic procedure for simulating the fracture process can be summarized as the following:

(1) The specimen of a heterogeneous material is discretized to a mesh system composed of many elements.

(2) The mechanical parameters are assumed to obey a stochastic distribution, such as Weibull's distribution.

(3) The displacement control method is adopted to simulate the loading process of a rock specimen in order to obtain the complete stress-strain curve of a rock failure process. The maximum displacement of each load step is usually limited to 1×10^{-6} to 1×10^{-7} m.

(4) A given yield criterion, such as the maximum tension yield criterion and the Mohr-Coulomb yield criterion, is used to determine whether the element yields or not.

(5) If there are failed elements in the loading process, the unbalanced force produced by the yielded elements will be transferred to their neighboring elements, causing a system stress readjustment. If some new elements failed in the stress adjustment process, their Young's modulus will be reduced according to the degradation parameter and the stress readjustment will continue until there is no other element failure occurrence in the system.

(6) So as to achieve stress adjustment equilibrium, the next load step will be applied to the system. FLAC3D (Fast Lagrangian Analysis of Continua in 3 Dimensions), produced by the Itasca Consulting Group¹³, is a well-known numerical software for engineering mechanics. FLAC3D is highly suitable for the large-strain simulation of continua because of the incorporation of an explicit solution scheme that provides stable solutions to unstable physical processes. Furthermore, users can incorporate their own constitutive models by writing a function using a built-in programming language. Doing so

results in an easy means of enhancing the program and, thus, assists in solving complex problems in engineering mechanics. Therefore, in this work, FLAC3D has been adopted for the implementation of the fracturing process simulation of a heterogeneous material.

Simulation of rock failure in true triaxial test: The true triaxial test is primarily concerned with compressing the rock specimen with three different principal stresses, namely, $\sigma_1 > \sigma_2 > \sigma_3$. According to the heterogeneous material model proposed in the previous section, a series of numerical models, with the same elemental seed parameters and same homogeneous index ($m = 5$), were built to conduct triaxial compressive strength tests. The plane rock specimens were all of 100 mm width, 100 mm length and 200 mm height with a total of 5,400 elements. The constant strain rate loading control method was adopted in order to simulate the loading process and the displacement increment was 5×10^{-7} m for each step. The load path is loading initial stress ($\sigma_1, \sigma_2, \sigma_3$) on the rock specimen and then unloading σ_3 to 0 MPa and finally, increasing σ_1 until the rock specimen fails. The initial stress of σ_1, σ_2 and σ_3 are, respectively 40, 30 and 10 MPa. The material properties of the rock specimen used in the simulation were obtained by both the rock test and the back-analysis and are shown in Table-1.

TABLE-1 MATERIAL PARAMETERS OF ROCK SPECIMENS			
Material parameters	Value	Material parameters	Value
Young's modulus	68 (Gpa)	Residual cohesion	3 (Mpa)
Poisson's ratio	0.24	Maximum friction angle	49 (°)
Tensile strength	8 (Mpa)	Homogeneous index	5.0
Initial cohesion	31 (Mpa)	Degradation parameter	0.98

Fig. 2 shows the rock fracture process for the rock specimens' two sides. It can be seen that with additional loads, more and more elements fail, which causes the rock specimen to finally fail and the V crack suddenly appear on the b side. It also reveals that failure elements focus on the crack result in brittle failure. In Fig. 3, the simulation result is compared with the experimental result and they agree well with one another. It can be concluded that the simulated method is feasible for simulating the failure process of rock triaxial test.

The distribution of energy release rate in different loading steps is shown in Fig. 4 and it is in accordance with failed elements focusing on cracking. In all the fracturing processes, the energy release distributes throughout the model and then gradually focuses on cracking. Finally, there are 20 loading steps from the surrender to overall failure and the greatest energy release occurs in this period. This indicates that the specimen broke heavily.

The energy release curve and the stress-strain curve is shown in Fig. 5. In the pre-peak stage, there are a few destructive elements with a few energy releasing. In the peak stage, most failure elements correspond with a great deal of released energy, which causes the rock specimens failure and the bearing capacity suddenly drops.

Conclusion

The proposed numerical model, which was based upon the improved CWFS constitutive theory and the local energy

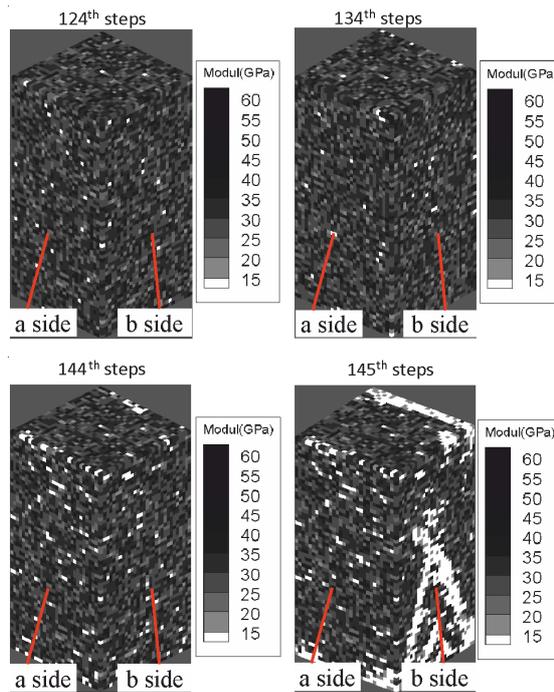
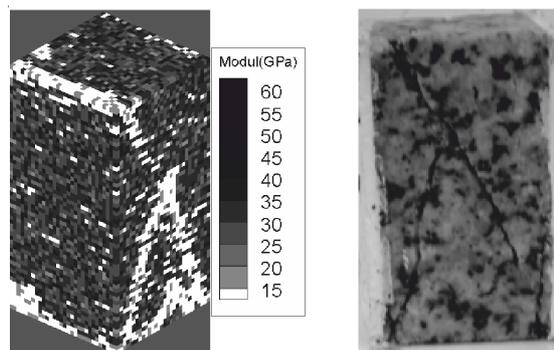


Fig. 2. Failure process of rocks



(a) Simulation result (b) Experimental result

Fig. 3. Comparison between the experimental result and the simulation result

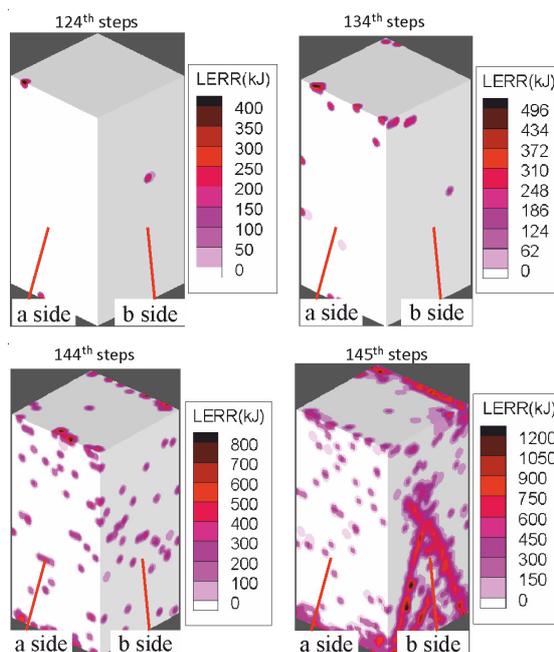


Fig. 4. Distribution of energy release rate in different loading steps

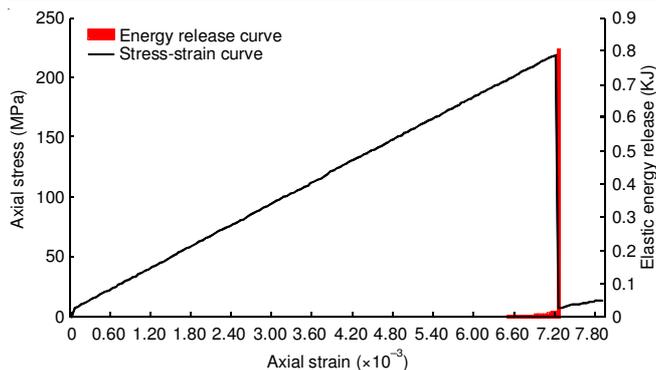


Fig. 5. Stress-strain curve and the release of energy curve

release rate theory, can accurately simulate the complete fracturing process and the energy release process for a heterogeneous material, such as a rock, under triaxial loading condition. This model can conveniently incorporate the heterogeneity of the material's microstructure. It additionally provides a powerful means for studying the macroscopic fracturing behaviour of a heterogeneous material. The simulated results of the rock triaxial compressive test agree well with the experimental results. This proves that the simulated method is suitable for simulating the triaxial compressive experiment. In addition, the second principal stress significantly influences the compressive strength of rock.

ACKNOWLEDGEMENTS

The authors thank the National Natural Science Foundation of China for financial support (Grant No. 51369007).

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