



Study on Solution Space for Visual Positioning Algorithm from Linear Features

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New solution method is presented for vision positioning method from straight lines intersecting at two spatial points. The solution method has fast calculation and can guarantee the uniqueness for solution. Application of this method is to satisfy precondition of monotony. This paper studies the geometry of space and gives sufficient proof of the monotonic the algorithm satisfies in the geometric range. In the spatial distribution of linear feature, geometric space can guarantee the convergence of the method. Thus, it can provide sufficient theoretical basis for the practical application of designed iterative method. Meanwhile, it can ensure the calculation speed and the uniqueness of solving methods. Simulation results show that iterative method can converge to right solution in the geometric space and has rapid iteration speed.

Keywords: Visual positioning, Solution space, Linear feature.

INTRODUCTION

In computer vision, visual positioning is to estimate three dimensional attitude of an object with respect to camera. It has become a hot research topic for vision positioning based on line features for computer vision communities since 2002. Visual positioning algorithm from single camera has wide varieties of applications, such as obstacle avoidance, industrial production and robot self-positioning¹⁻⁴.

For vision positioning with line correspondences, minimum of three straight lines is required to solve the problem of pose estimating, where not two of which are parallel. Then three lines can produce three non-linear equations, so the vision positioning question is converted into solving a non-linear equations set. Currently, approaches to solve this question fall into two classes: closed-form methods and iterative methods⁵.

For iterative algorithms, Yuan⁶ has used Newton's method to determine the position of 3D object with respect to a camera. Liu *et al.*⁷ have examined alternative iterative methods to solve the vision pose. Phong *et al.*⁵ used trust region optimization algorithm to solve orientation and location in 1995. Christy and Horaud⁸ have calculated viewing parameters from iterative method, which started with a solution near to true solution. Iterative approaches have two shortcomings, which need a good initial estimation of the true solution and be time-consuming.

A new method for estimating object vision pose for three non-coplanar lines which intersect at two points has been proposed by Qin and Zhu⁹. We have presented a new iterative method which is based on the geometry and step acceleration method for the solution of this method. This method gives insight into the geometric characteristics of the problem. The search process satisfies convergence characteristic, so the solution method can avoid be trapped in local minima. Meanwhile, the solution method is free of choosing original estimation value and it can find the true value ever if the initial estimation is far from true value.

The premise for running of our iterative method is existing convergence region. For this reason, we will give detailed proof for convergence of solution space. The value of the proof is to provide the theoretical basis for iterative solution method and guide the application of the solution method. We have found one region where iterative method satisfies converge in paper¹⁰⁻¹². In recent research work, we have found another region where this iterative method can also satisfy convergence. The work of this paper is to present and proof this region.

Vision positioning method from three lines: We consider pin-hole camera model. The intrinsic camera parameters are assumed to be known. Fig. 1 is perspective projection of three general lines intersecting at two points in monocular vision.

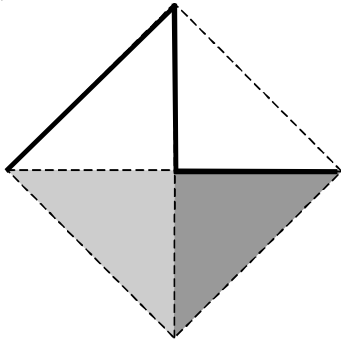


Fig. 1. Visual positioning model for space lines intersecting at two points

Line L_i , image line l_i and the origin of the camera frame are constrained to lie in the same plane as shown in Fig. 2 by perspective projection model. This plane is called the explanation plane. We know model lines satisfy conditions which are $|p_2p_3| = a$, angle between line L_1 and line L_2 is α , angle between line L_3 and line L_2 is β and angle between line L_1 and line L_3 is γ .

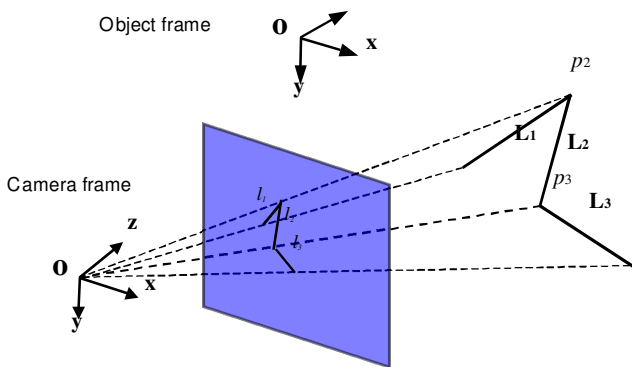


Fig. 2. Perspective projection for three line features intersecting at two points in monocular vision

Vector $N_i = (N_{i1}, N_{i2}, N_{i3})^T$ normal to the explanation plane can be computed as the cross product of vector of o and vector v_i , it can be get that $o_i = (x_i, y_i, f)^T$, $v_i = (-b_i, a_i, 0)^T$. Then $N_i = v_i \times o_i = (a_i f, f_i f, c_i)^T$ can be computed.

Because line L_1 lies in the interpretation plane formed by the image line l_1 and the optical center, the angle between line L_1 and line L_2 is α and the norm of the vectors of line L_1 is unit, we can obtain equation as following:

$$A_1 N_{11} + B_1 N_{12} + C_1 N_{13} = 0, A_1 A_2 + B_1 B_2 + C_1 C_2 = \cos \alpha, A_1^2 + B_1^2 + C_1^2 = 1 \quad (1)$$

For line L_3 , because line L_3 lies in the interpretation plane formed by the image line l_3 and the optical center, the angle between line L_3 and line L_2 is β and meanwhile norm of the vectors of line L_3 is unit, we can have equation as following:

$$A_3 N_{13} + B_3 N_{32} + C_3 N_{33} = 0, A_3 A_2 + B_3 B_2 + C_3 C_2 = \cos \beta, A_3^2 + B_3^2 + C_3^2 = 1 \quad (2)$$

From condition that the angle between line L_1 and line L_3 is γ , we get:

$$A_1 A_3 + B_1 B_3 + C_1 C_3 = \cos \gamma \quad (3)$$

Substituting eqns. 1 and 2 into 3, we can obtain a new expression.

Direction vectors of line L_2 can be expressed by points p_2 and p_3 :

$$L_2 = (A_2, B_2, C_2)^T = (k_3 x_3 - k_2 x_2, k_3 y_3 - k_2 y_2, k_3 z_3 - k_2 z_2)^T \quad (4)$$

Substitute eqn. 4 into 3, equation about k_2 and k_3 can be obtained.

From condition $|p_2p_3| = a$, we get:

$$(k_3 z_3 - k_2 z_2)^2 + (k_3 y_3 - k_2 y_2)^2 + (k_3 x_3 - k_2 x_2)^2 = a^2 \quad (5)$$

So far we get two equations about two unknowns. If we substitute one into another, we can have one new expression. We assume the new equation is $f(k_2) = 0$. If we obtain the value of k_2 , thus $k_3, A_1, B_1, C_1, A_2, B_2, C_2, A_3, B_3, C_3$ can be determined from corresponding equations, then the coordinates of p_2 and p_3 and the director vectors of line L_1, L_2, L_3 in camera frame can be computed. We propose one solution method which is based on geometry and step acceleration method to solve k_2 .

Solution approach: From above analysis, we obtain one expression F which is in terms of one variant k_2 and this expression is complex to solve. To solve this problem, we present a new method to compute based on geometry and step acceleration method.

The optical center is supposed to O . S_1 is explanation plane formed by image line l_1 and O , S_2 is plane formed by image line l_2 and O . S_3 is plane formed by image line l_3 and O . J_3 is the intersection line which is formed by the explanation plane S_1 and the explanation plane S_2 . J_4 is the intersection line formed by plane S_2 and S_3 (Fig. 3).

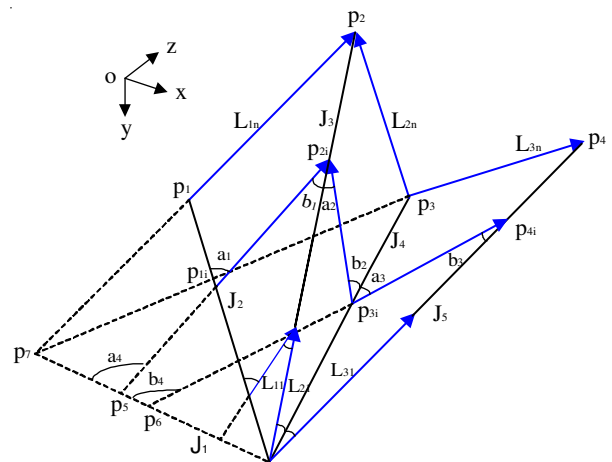


Fig. 3. Iterative solution process

Correct value of k_2 is searched based on geometry and step acceleration method (Fig. 3), we select a point p_{2i} on line J_3 which is near the optical center firstly. Second, we continue to search point p_{3i} at a distance of a from point p_{2i} on line J_4 . Line L_{2i} is formed by point p_{2i} and point p_{3i} . Third, we find a line L_{1i} on explanation plane S_1 which forms an angle of α with line L_{2i} . Fourth, we find the line L_{3i} on explanation plane S_3 which makes an angle of β with line L_{2i} . Angle γ_i is formed by line L_{1i} and line L_{3i} . Angle between line L_{1i} and line L_{3i} becomes larger with the increase of the distance between point p_{2i} and the optical center of camera. Point p_{2i} and three lines $L_{1i}, L_{2i},$

L_{3i} reach the right position when the angle satisfies the known value γ . Then the whole iterative process finishes.

Solution space for visual positioning method and proof: (Fig. 3) we do some assumption. The angle between line L_1 and line J_2 is a_1 , angle between line L_1 and line J_3 is b_1 and angle between line L_2 and line J_3 is a_2 . Angle between line L_2 and line J_4 is b_2 , angle between line L_3 and line J_4 is a_3 and angle between line L_3 and line J_5 is b_3 . It is assumed that angle between line L_1 and line J_1 is a_4 , the angle between line L_3 and line J_1 is b_4 . The angle between explain plane S_1 and S_2 is φ_1 , the angle between explain plane S_2 and S_3 is φ_2 and the angle between plane S_1 and plane S_3 is φ_3 . At regions as following, the iterative method satisfies convergence and can find the right solution.

Solution space for visual positioning method:

$$(1) \cos(\varphi_1) > 0, 0 < a_2 < \frac{\pi}{2}, \pi < b_1 < \frac{3\pi}{2}, \frac{3\pi}{2} < b_1 + \gamma_1 < \frac{5\pi}{2}$$

$$(2) \frac{\pi}{2} < \varphi_2 < \pi, 0 < a_3 < \frac{\pi}{2}, 0 < b_2 < \frac{\pi}{2}, \frac{\pi}{2} < a_3 + \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right) < \frac{3\pi}{2}$$

$$(3) 0 < \varphi_3 < \frac{\pi}{2}, 0 < a_4 < \frac{\pi}{2}, 0 < b_4 < \frac{\pi}{2}, \tan(a_4) \cot(b_4) > \frac{1}{\cos(\varphi_3)}$$

Three conditions (1) (2) (3) are simultaneously satisfied.

Lemma 1: (Fig. 4) with the increase of variable k_2 , the angle b_1 will increase correspondingly in the case $\cos(\varphi_1) > 0, 0 < a_2 < \pi/2, \pi < b_1 < 3\pi/2, 3\pi/2 < b_1 + \gamma_1 < 5\pi/2$.

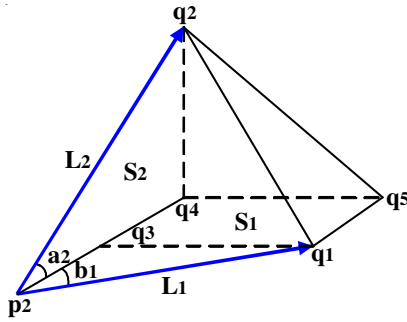


Fig. 4. Change trend analysis for line L_1 with the development of k_2

Proof: As shown in Fig. 3, it is obvious to see that as independent variable k_2 increase, angle a_2 and angle b_2 will increase correspondingly.

Fig. 4, the length of line segment p_2q_2 and p_2q_1 are assumed to be one and angle φ_1 is invariant during iterative process. It is supposed to drop perpendicular line from point q_1 to line J_3 and intersect J_3 at q_3 . Dropping perpendicular line from point q_2 to line J_3 and intersecting J_3 at q_4 . Drawing line q_4q_5 parallel to line q_1q_3 from point q_4 . Furthermore, length of q_1q_3 is equal to length of q_4q_5 . Connecting point q_1 with point q_2 , point q_1 with point q_5 and point q_2 with point q_5 .

In, $\Delta q_2p_2q_4$,

$$p_2q_4 = p_2q_2 \cos(a_2) = \cos(a_2), q_2q_4 = p_2q_2 \sin(a_2) = \sin(a_2) \quad (6)$$

In, $\Delta q_1p_2q_3$,

$$p_2q_3 = p_2q_1 \cos(b_1) = \cos(b_1), q_2q_3 = p_2q_1 \sin(b_1) = \sin(b_1) \quad (7)$$

Since $q_1q_3 \perp J_3, q_4q_5 \perp J_3, q_1q_5 \parallel q_3q_4, q_1q_5 = q_3q_4$, then $q_1q_3q_4q_5$ satisfies to be rectangle. It can be get as following:

$$q_1q_5 = q_3q_4, q_4q_5 = q_1q_3 \quad (8)$$

Thus,

$$q_1q_5 = p_2q_4 - p_2q_3 = \cos(a_2) - \cos(b_1) \quad (9)$$

In $\Delta q_2q_4q_5$, $q_2q_4 \perp J_3$ and $q_4q_5 \perp J_3$, it is obvious to get:

$$\angle q_2q_4q_5 = \varphi_1 \quad (10)$$

Application the law of cosines,

$$q_2q_5 = \sqrt{q_2q_4^2 + q_4q_5^2 - 2q_2q_4q_4q_5 \cos(\varphi_1)} = \sqrt{\sin(a_2)^2 + \sin(b_1)^2 - 2\sin(a_2)\sin(b_1)\cos(\varphi_1)} \quad (11)$$

Since $q_3q_4 \perp q_2q_4, q_3q_4 \perp q_4q_5$, we have $q_3q_5 \perp \Delta q_2q_4q_5$. In the further step, we can obtain $q_3q_4 \perp q_2q_5$. Since $q_1q_5 \parallel q_3q_4, q_3q_4 \perp q_2q_5$, we have $q_1q_5 \perp q_2q_5$. Therefore, $\Delta q_1q_2q_5$ is a right triangle.

It is satisfied in $\Delta q_1q_2q_5$,

$$q_1q_2 = \sqrt{(q_1q_5)^2 + (q_2q_5)^2} = \sqrt{2 - 2\sin(a_2)\sin(b_1) - 2\cos(a_2)\cos(b_1)\cos(\varphi_1)} \quad (12)$$

In, $\Delta q_2p_2q_1$,

$$\cos \angle q_2p_2q_1 = \frac{q_2p_2^2 + q_1p_2^2 - q_2q_1^2}{2q_2p_2q_1p_2} = \cos(a_2)\cos(b_1)\cos(\varphi_1) + \sin(b_1)\sin(a_2) \quad (13)$$

Cosine of the angle between line L_1 and line L_2 is assumed to satisfy:

$$F = \cos(a_2)\cos(b_1)\cos(\varphi_1) + \sin(b_1)\sin(a_2) \quad (14)$$

Eqn. 14 can be expressed as following:

$$F = \sqrt{1 - (\sin(a_2))^2 (\sin(\varphi_1))^2} \sin(b_1 + \gamma_1) \quad (15)$$

where, $\gamma_1 = \arctan\left(\frac{\cot(a_2)}{\cos(\varphi_1)}\right)$.

As independent variable k_2 increases, a_2 will increase correspondingly in the iterative process. Then $\sqrt{1 - (\sin(a_2))^2 (\sin(\varphi_1))^2}$ will decrease.

As independent variable k_2 increases, a_2 will increase correspondingly in the iterative process. Then $\cot(a_2)$ will

decrease. For $\cos(j_1) > 0$ and $\gamma_1 = \arctan\left(\frac{\cot(a_2)}{\cos(\varphi_1)}\right)$, then g_1

will decrease as independent variable k_2 increases. Because $3\pi/2 < b_1 + \gamma_1 < 5\pi/2$, thus $\sin(b_1 + \gamma_1)$ will decrease with γ_1 decreasing.

Conclusion can be easily get that if function $F = \sqrt{1 - (\sin(a_2))^2 (\sin(\varphi_1))^2} (\sin(b_1 + \gamma_1))$ should remain constant, then $\sin(b_1 + \gamma_1)$ will decrease. Because $3\pi/2 < b_1 + \gamma_1 < 5\pi/2$ and γ_1 decreases, thus b_1 will increase. Therefore, as independent variable k_2 increases, then b_1 will increase in the case $\cos(\varphi_1) > 0, 0 < a_2 < \pi/2, \pi < b_1 < 3\pi/2, 3\pi/2 < b_1 + \gamma_1 < 5\pi/2$. Q.E.D.

Lemma 2: Fig. 5, with the increase of independent variable k_2 , a_3 should increase correspondingly in the case $\pi/2 < \varphi_2 < \pi, 0 < a_3 < \pi/2, \pi/2 < a_3 + \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right) < \frac{3\pi}{2}$.

Proof: Cosine of the angle between line L_2 and line L_3 is assumed to satisfy the following expression:

$$F = \cos(a_3) \cos(b_2) \cos(\varphi_2) + \sin(b_2) \sin(a_3) \quad (16)$$

Eqn. 16 is arranged into:

$$F = \sqrt{1 - (\sin(b_2))^2 (\sin(\varphi_2))^2} (\sin(a_3 + \gamma_2)) \quad (17)$$

where, $\gamma_2 = \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right)$.

With the increase for independent variable k_2 , b_2 will increase correspondingly in the iterative process, then $\sqrt{1 - (\sin(b_2))^2 (\sin(\varphi_2))^2}$ will decrease.

With the increase for independent variable k_2 in the iterative process, b_2 will increase correspondingly, thus $\cot(b_2)$ will decrease. For $\pi/2 < \varphi_2 < \pi$, thus $\cos(\varphi_2) > 0$. Because $\gamma_2 = \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right)$, then γ_2 will decrease.

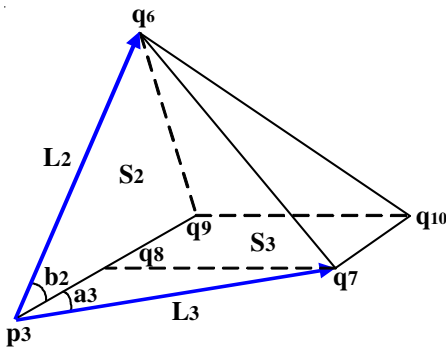


Fig. 5. Change trend analysis for line L_3 with the development of k_2

Since $\pi/2 < a_3 + \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right) < \frac{3\pi}{2}$, then $\pi/2 < a_3 + \gamma_2 < 3\pi/2$ and $\sin(a_3 + \gamma_2)$ will decrease with γ_2 increasing.

Function $F = \sqrt{1 - (\sin(b_2))^2 (\sin(\varphi_2))^2} (\sin(a_3 + \gamma_2))$ should remain constant. As independent variable k_2 increases, $\sqrt{1 - \sin(b_2) \sin(\varphi_2)^2}$ will decrease, that is $\sin(a_3 + \gamma_2)$ will increase.

It can be obtained that a_3 will decrease because $\pi/2 < a_3 + \gamma_2 < 3\pi/2$.

Therefore, with the increase of independent variable k_2 , a_3 will increase correspondingly in the case $\pi/2 < \varphi_2 < \pi, 0 < a_3 < \pi/2, 0 < b_2 < \pi/2, \pi/2 < a_3 + \arctan\left(\frac{\cot(b_2)}{\cos(\varphi_2)}\right) < \frac{3\pi}{2}$. Q.E.D.

Theorem: Fig. 6, with the increase of independent variable k_2 , angle between line L_1 and line L_3 should increase correspondingly in the case $0 < \varphi_3 < \pi/2, 0 < a_4 < \pi/2, 0 < b_4 < \pi/2, \tan(a_4) \cot(b_4) > \frac{1}{\cos(\varphi_3)}$.

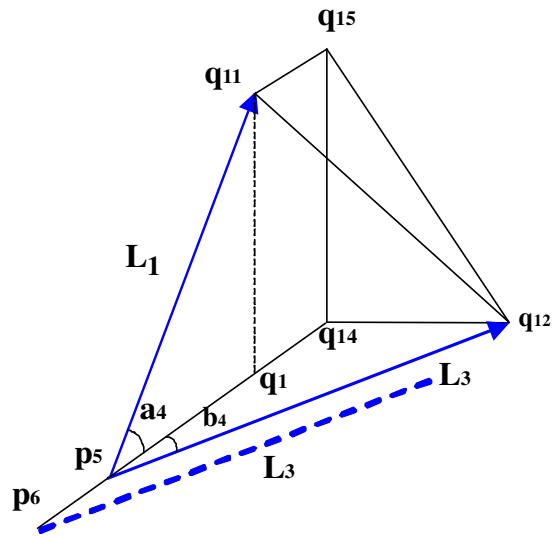


Fig. 6. Change trend analysis for angle between line L_1 and line L_3 with the development of k_2

Proof: Cosine of the angle between line L_1 and line L_3 is assumed to satisfy:

$$F = \cos(a_4) \cos(b_4) \cos(\varphi_3) + \sin(b_4) \sin(a_4) \quad (18)$$

Eqn. 18 is arranged into:

$$F = \sqrt{1 - (\sin(a_4))^2 (\sin(\varphi_3))^2} (\sin(b_4 + \gamma_3)) \quad (19)$$

where, $\gamma_3 = \arctan\left(\frac{\cot(a_4)}{\cos(\varphi_3)}\right)$.

Considering $\tan(b_4) \cot(a_4) > \frac{1}{\cos(\varphi_3)}$, we have

$$\frac{\cot(a_4)}{\cos(\varphi_3)} < \tan\left(\frac{\pi}{2} - b_4\right). \text{ Since } \tan(\gamma_3) = \frac{\cot(a_4)}{\cos(\varphi_3)}, \text{ we have}$$

$$\tan(\gamma_3) < \tan\left(\frac{\pi}{2} - b_4\right) \text{ and } 0 < b_4 + \gamma_3 < \pi/2.$$

As the same as proof process of Lemma 1, it can be readily get that angle b_1 will increase correspondingly in the case $\cos(\varphi_1) > 0, 0 < a_2 < \pi/2, \pi < b_1 < 3\pi/2, 3\pi/2 < b_1 + \gamma_1 < 5\pi/2$ as independent variable k_2 increases. It is considered that $a_4 = b_1 + \angle p_2Op_5$ and $\angle p_3Op_6$ do not change in iterative process, thus it is easy to obtain angle a_4 will increase correspondingly in this case as independent variable increases.

Therefore, $\sqrt{1 - (\sin(a_4))^2 (\sin(\varphi_3))^2}$ will decrease for increase of a_4 .

It can be obtained that $\cot(a_4)$ will decrease as A_4 increases in iterative process. Mean while, for $0 < \varphi_3 < \pi/2$, we have $\cos(\varphi_3) > 0$. Then γ_3 will decrease considering

$$\gamma_3 = \arctan\left(\frac{\cot(a_4)}{\cos(\varphi_3)}\right).$$

From the proof of Lemma 2, we can readily have that angle a_3 will decrease correspondingly in the case $0 < \varphi_2 < \pi/2, 3\pi/2 < a_3 < 2\pi, 3\pi/2 < a_3 + \gamma_2 < 5\pi/2$ as independent variable k_2 increases.

Considering $b_4 = a_3 + \angle p_3Op_6$ and $\angle p_3Op_6$ do not change during the iterative process. Thus it is obvious to get as independent variable k_2 increases; angle b_4 will decrease correspondingly in above case.

Noting $0 < \gamma_3 + b_4 < \pi/2$ and angle b_4 and γ_3 will decrease as independent variable k_2 increases in some case, it can be obtained easily that $\sin(b_4 + \gamma_3)$ will decrease correspondingly with the increase of independent variable k_2 . Then we have

$$F = \sqrt{1 - (\sin(a_4))^2 (\sin(\varphi_3))^2} (\sin(b_4 + \gamma_3)) \text{ will decrease.}$$

Thus we can draw the conclusion that cosine of angle between line L_1 and line L_3 will decrease as independent variable k_2 increases in some case. So, as independent variable k_2 increase, angle between line L_1 and line L_3 will increase correspondingly in the case (1) (2) (3). Q.E.D.

RESULTS AND DISCUSSION

We do the following instance assumption for the length and angle of visual localization model for space lines intersecting at two points in Fig. 1. Simulation experiments were carried out in the case camera view angle is $36^\circ \times 36^\circ$. It is assumed that the length of each line segments is 30 cm for three space lines intersecting at two points. The angel between each two line segments is 60° in three line segments.

The angle between the first line and third lines is monotone increasing in the geometric space we study in this paper with the increase of distance between initial iteration point and the camera optical center in monocular vision positioning method. Convergence curve is shown in Fig. 7 that iterative solution method converges to the correct answer. The abscissa represents the distance between initial iteration point and camera optical center. The ordinate represents the angle between first line and third line. The solution process finishes when the angle is achieved at and the iterative method converges to the correct solution.

The results are shown in Fig. 8 if we do the analysis for running time of iterative methods in solving geometric space studied in this paper. Fig. 8 shows the average consumption time in each test point for the two solution method. Simulation results show that the iterative method for solving geometric space studied in this paper has faster iterative speed and shorter operation time relative to the iteration method in the reference⁷.

Conclusion

Numerical solution method is not easy to guarantee the uniqueness of numerical solution method in the optimization

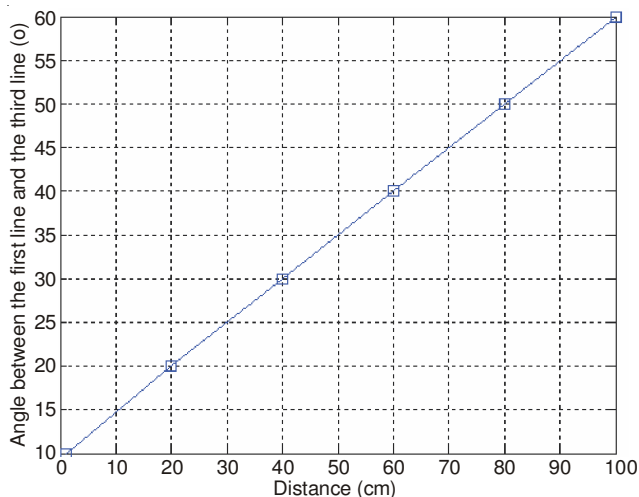


Fig. 7. Convergence diagrammatic sketch of iteration method in solving geometric space

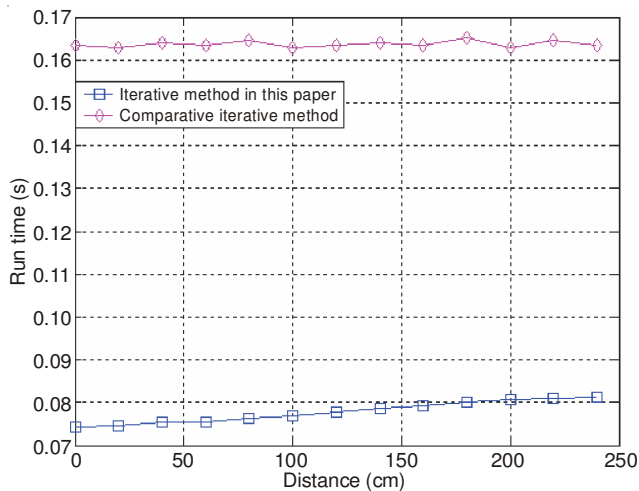


Fig. 8. Comparison picture of running time for iterative methods

process although the visual position has the advantages of high measuring precision. At the same time, the iterative time is long. A new vision iterative method has been proposed based on geometrical meaning and step acceleration method for three space lines intersecting at two points of visual positioning method. This method is fast and easy to guarantee the uniqueness of the solution. It can avoid falling into local minima. The application premise of this method is to satisfy monotonic so as to find the geometric space the solution can satisfy the monotonic and ensure vision measurement method converges to the correct solution. We propose detailed proof process for the monotonic of solution geometric space for iterative method in this paper. The value of research of geometry space is to provide theoretic base for practical application of iterative method and direct the application of visual measurement algorithm.

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REFERENCES

1. J.H. Qiu, M.X. Shen, J.H. Cong and L.G. Li, *J. Zhejiang Forestry College*, **27**, 65 (2010).
2. X.Z. Tang, Z.M. Sun and X.L. Jia and D. He, *Geogr. Spatial Inform.*, **10**, 100 (2012).
3. Y.X. Han, Z.S. Zhang and M. Dai, *Optics Precision Eng.*, **19**, 1110 (2011).
4. S.J. He, Z.Y. Liu and J.Q. Shi, *Computer Applications*, **32**, 2556 (2012).
5. T.Q. Phong, R. Horaud, A. Yassine and P.D. Tao, *Int. J. Comput. Vis.*, **15**, 225 (1995).
6. J.S.-C. Yuan, *IEEE Trans. Robot. Autom.*, **5**, 129 (1989).
7. Y. Liu, T.S. Huang and O.D. Faugeras, *IEEE Trans. Pattern Anal. Mach. Intell.*, **12**, 28 (1990).
8. S. Christy and R. Horaud, *Comput. Vis. Image Underst.*, **73**, 137 (1999).
9. L.J. Qin and F. Zhu, IEEE International Conference on Information and Acquirement, p. 26 (2006).
10. L.J. Qin and Y.L. Hu, *Adv. Mater. Res.*, **186**, 650 (2011).
11. L.J. Qin, Y.L. Hu, Y.Z. Wei, Y. Zhou and H. Wang, Chinese Control and Decision Conference, pp. 258-263 (2008).
12. L.J. Qin, *J. Shenyang Ligong Univ.*, **32**, 1 (2013).