



## Research on Energy Loss Eoefficient of Sudden Varying Tube by Simulation

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Sudden-enlargement tube and sudden-reduction tube have important applications in reality lives. Many researches have studied the characteristics of sudden-enlargement tube flows and sudden-reduction tube flows and many valuable results have also been obtained. But the valuable results only illuminate the relationships between energy loss coefficient and impacting factors, empirical expression about energy loss coefficient cannot be achieved. In this paper, empirical expression about energy loss coefficient was studied and was obtained by using simulation method. It could be concluded that the energy loss coefficient was mainly dominated by the contraction ratio. The less the contraction ratio is, the larger is the energy loss coefficient. When Reynolds number is more than 105, Reynolds number has little impacts on energy loss coefficient.

**Keywords:** Transportation in tube, Empirical expression, Simulation, Energy loss coefficient, Sudden varying tube.

### INTRODUCTION

Sudden enlargement and sudden reduction flows have been applied widely in fluid measurement. In recent years, with the development of techniques, sudden enlargement and sudden reduction forms pipes were used to dissipate energy in tunnel<sup>1</sup>. The orifice plate or plug, as a kind of energy dissipater with sudden reduction and sudden enlargement forms, has been used in the hydropower projects due to its simple structure, convenient construction and high energy dissipation ratio<sup>2</sup>. As early as 1960s, a plug dissipater, similar to orifice plate in energy dissipation mechanism, with the energy dissipation ratio of over 50 %, was used in the flood discharge tunnel of the Mica dam in Canada<sup>3</sup>. In 2000, a three-stage sharp-edged orifice plate was applied in the Xiaolangdi Projects in China, gets the energy dissipation ratio of about 44 %<sup>4</sup>. The practical application has proved that it is entirely feasible to utilize sudden enlargement and sudden reduction forms to dissipate flow's tremendous energy in hydropower project. So it is important to carry out research on related hydraulics problems of sudden enlargement tube and sudden reduction tube is necessary. Wang and Yue<sup>5</sup> presented an approximate method to calculate the energy loss coefficients of both sudden reduction and sudden enlargement for an orifice plate by means of the physical model experiments and the coefficients are  $\zeta_1 = (1/C_c - 1)^2$  for sudden reduction flows and  $\zeta_2 = (1/C_c - \beta^2)^2$

for sudden enlargement flows, respectively, where  $C_c = A_c/A_d$ , is the area ratio of the reduction section ( $A_c$ ) of the flow through the orifice plate to the orifice section ( $A_d$ ) and  $\beta = d/D$ . The local resistance coefficients are the function of  $A_c$  and  $\beta$ . However, there were, at least, two factors not to be considered in their method, one is the difficulty to determine the size and position of section  $A_c$  so that one can not compute  $C_c$ ; the other is the effects of the hydraulic parameters on  $A_c$  since it varies with different flow conditions. So it is necessary to study the energy loss coefficients of both sudden reduction and sudden enlargement tube. The purposes of the present work, therefore, are to investigate the effects of the geometric parameters, *i.e.*, the contraction ratio and flow parameters, *i.e.*, Reynolds number, on the energy loss characteristics of sudden reduction and sudden enlargement tube.

**Definition of energy loss coefficient:** Sudden enlargement flows and sudden reduction flows are shown in Figs. 1 and 2, respectively. In Figs. 1 and 2, section 1-1, where flows are undisturbed, is located before sudden enlargement tube or sudden reduction tube; section 2-2, where flows already recover, is located after sudden enlargement tube or sudden reduction tube. The energy equation of the flow between section 1-1 and section 2-2 can be given<sup>6</sup>:

$$z_1 + \frac{p_1}{\gamma} + \frac{\alpha_1 u_1^2}{2g} = z_2 + \frac{p_2}{\gamma} + \frac{\alpha_2 u_2^2}{2g} (\xi_f + \xi_1) \frac{u_1^2}{2g} \quad (1)$$

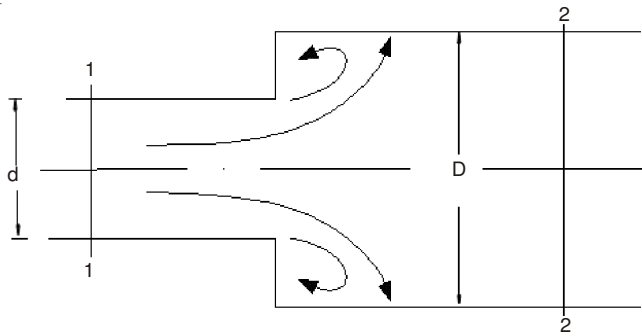


Fig. 1. Sudden enlargement tube flows

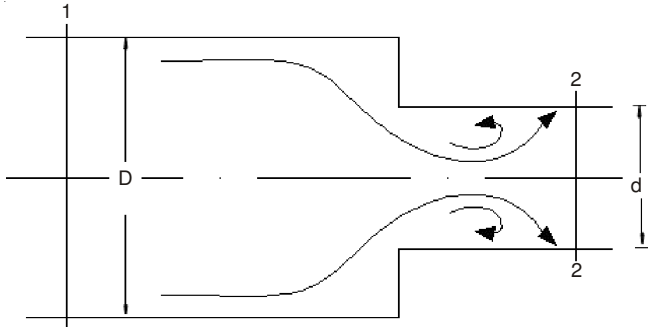


Fig. 2. Sudden reduction tube flows

where  $z_1$  and  $z_2$  are the elevation heads at sections 1-1 and 2-2, respectively;  $p_1$  and  $p_2$  are the average pressures at sections 1-1 and 2-2, respectively;  $u_1$  and  $u_2$  are the average velocities at sections 1-1 and 2-2, respectively;  $\alpha_1$  and  $\alpha_2$  are the kinetic energy correction factors at sections 1-1 and 2-2, respectively;  $\gamma$  is the specific weight of water;  $g$  is the acceleration of gravity; and  $\xi_f$  and  $\xi_l$  are the frictional energy loss and local energy loss coefficients, respectively. For the tube with the bottom slope of zero,  $z_1 = z_2$ . When the tube has same diameter  $D$ ,  $u_1 = u_2 = u$ , is the average velocity of the tube based on continuity equation. Let  $\alpha_1 = \alpha_2 = 1.0$ , neglect the effects of the frictional energy loss, *i.e.*,  $\xi_f = 0$  and let  $\xi_l = \xi$  and then eqn. 1 becomes:

$$k = \frac{p_1 - p_2}{0.5\rho u^2} = \frac{\Delta p}{0.5\rho u^2} \quad (2)$$

where  $k$  is energy loss coefficient. Many research results show that, for sudden enlargement tube flows and sudden reduction flows, energy loss coefficient  $k$  is related with Reynolds number and contraction  $d/D$ <sup>7,8</sup> ( $d$  and  $D$  are shown in Figs. 1 and 2, respectively). So eqn. 2 can be expressed:

$$k = f\left(\frac{d}{D, R_e}\right) \quad (3)$$

where  $R_e$  is Reynolds number.

## EXPERIMENTAL

**Numerical simulations model:** The RNG  $k$ - $\epsilon$  model was used to calculate the hydraulic parameters of the flow through sudden enlargement tube and sudden reduction tube, due to its suitability for simulating the flow inside large change boundary forms as well as its high precision and calculation stability. For the steady and incompressible flows, the governing equations of this model can be written as<sup>9,10</sup>:

Continuity equation:

$$\frac{\partial u_i}{\partial x_i} = 0, i = 1, 2 \quad (4)$$

Momentum equation:

$$u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left[ (v + v_t) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \right] i = 1, 2 \quad (5)$$

$k$ : equation:

$$u_i \frac{\partial k}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \alpha_k (v + v_t) \frac{\partial k}{\partial x_j} \right] + \frac{1}{\rho} G_k - \epsilon \quad i = 1, 2 \quad (6)$$

$\epsilon$ : equation:

$$u_i \frac{\partial \epsilon}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \alpha_\epsilon (v + v_t) \frac{\partial \epsilon}{\partial x_j} \right] + \frac{1}{\rho} C_1^* G_k \frac{\epsilon}{k} - C_2 \frac{\epsilon^2}{k} \quad i = 1, 2 \quad (7)$$

where  $x_i$  ( $= x, y$ ) are the coordinates in longitudinal and transverse directions, respectively;  $u_i$  ( $= u_x, u_y$ ) are the velocity components in  $x$  and  $y$  directions, respectively;  $\rho$  is the density of water;  $p$  is the pressure;  $v$  is the kinematic viscosity;  $v_t$  is the eddy viscosity and can be given by  $v_t = C_\mu (k^2/\epsilon)$ , in which  $k$  is the turbulence kinetic energy,  $\epsilon$  is the dissipation rate of  $k$  and  $C_\mu = 0.085$ . The other parameters are:

$$C_1^* = C_1 - \frac{\eta(1 - \eta/\eta_0)}{1 + \lambda\eta^3}$$

$$\eta = \frac{Sk}{\epsilon}, S = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

$$C_1 = 1.42, \eta_0 = 4.377, \lambda = 0.012$$

$$G_k = \rho v_t \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j}, C_2 = 1.68 \text{ and } \alpha_k = \alpha_\epsilon = 1.39.$$

The calculation boundary conditions are treated as follows: in the inflow boundary the turbulent kinetic energy  $k_{in}$  and the turbulent dissipation rate  $\epsilon_{in}$  can be defined as, respectively<sup>11</sup>:

$$k_{in} = 0.0144u_{in}^2, \epsilon_{in} = \frac{k_{in}^{1.5}}{(0.25D)} \quad (8)$$

where  $u_{in}$  is the average velocity in the inflow boundary. In the outflow boundary the flow is considered as developed fully. The wall boundary is controlled by the wall functions. And the symmetric boundary condition is adopted, that is, the radial velocity on symmetry axis is zero.

Because the sudden enlargement tube and sudden reduction tube have axial symmetry characteristics, three dimensional numerical simulations of flows can be simplified as two dimensional numerical simulations of tube flows. The 3-D coordinate axis of sudden enlargement tube is shown in Fig. 3. In this

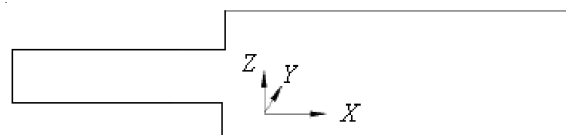


Fig. 3. 3-D coordinate axis of sudden enlargement tube

paper, flows' characteristics of plane XZ are researched; the characteristics of flows in plane XZ can represent the whole tube flows characteristics.

**RESULTS AND DISCUSSION**

**Verification for calculation results:** About the energy loss coefficient of sudden enlargement tube flows, Russell and Ball<sup>3</sup> conducted physical experiment. Fig. 4 is the comparison results between this paper's calculation data and Russell's experiment data. From Fig. 4, it can be learned that this paper's calculation results are coincided with those obtained by experiment. So it is feasible to research the characteristics of sudden enlargement tube flows and sudden reduction tube flows by using this paper's simulation method.

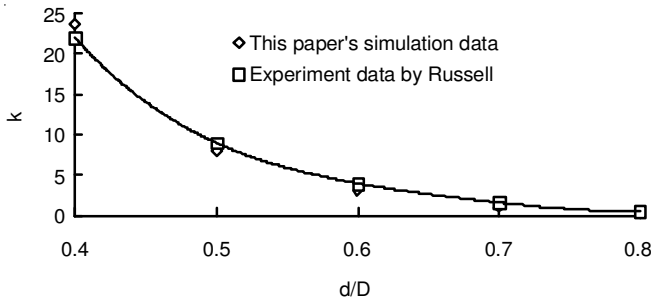


Fig. 4. Comparisons between experimental data and simulation data

**Effects of Reynolds number:** The data in Table-1 are calculation results about the effects of Reynolds number on energy loss coefficient when d/D is 0.5. From Table-1, the following conclusions can be obtained: when Reynolds number is less than 10<sup>5</sup>, energy loss coefficient k of sudden enlargement flows and sudden reduction flow increases slightly with the increase of Reynolds number, but when Reynolds number is more than 10<sup>5</sup>, Reynolds number has little effect on energy loss coefficient.

**Effects of contraction ratio:** Fig. 5 is streamline picture of sudden enlargement flows when d/D = 0.4, R<sub>e</sub> = 1.2 × 10<sup>5</sup>, Fig. 6 is streamline picture of sudden reduction flows when d/D = 0.8, R<sub>e</sub> = 1.2 × 10<sup>5</sup>. From Figs. 5 and 6, it can be learned that flow drastically changes in the vicinity of sudden enlargement and sudden reduction tube, which results in flow's energy loss. Fig. 7 is simulation results about sudden enlargement flows energy loss when R<sub>e</sub> = 1.2 × 10<sup>5</sup>. Fig. 8 is simulation results about sudden reduction flows energy loss when R<sub>e</sub> = 1.2 × 10<sup>5</sup>. Figs. 7 and 8 show that, for sudden enlargement tube flows and sudden reduction tube flows, energy loss coefficient decreases with the increase of contraction ratio. The clause deriving the above phenomenon is that: backflow area is main energy loss source, backflow region's length of sudden enlargement tube flow and sudden reduction tube flow decrease with the increase of contraction ratio. By fitting the curve in Fig. 7, empirical expression about energy loss coefficient of sudden enlargement tube flows can be obtained:

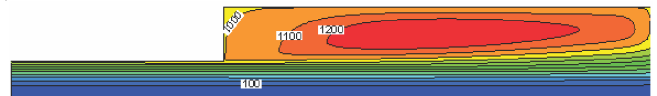


Fig. 5. Streamline for sudden enlargement flows (d/D = 0.4, R<sub>e</sub> = 1.2 × 10<sup>5</sup>)

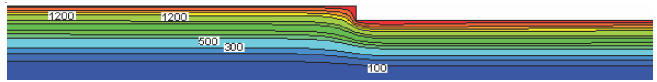


Fig. 6. Streamline for sudden reduction flows (d/D = 0.8, R<sub>e</sub> = 1.2 × 10<sup>5</sup>)

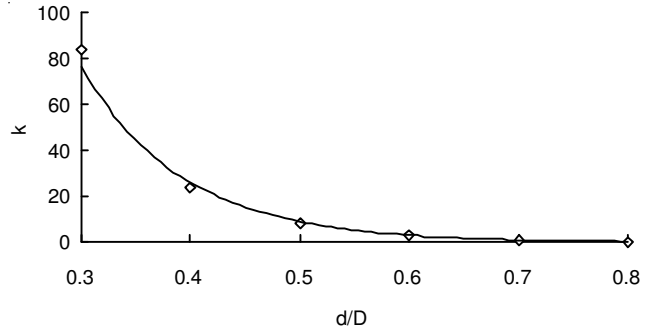


Fig. 7. Relationships between k and d/D for sudden enlargement flows

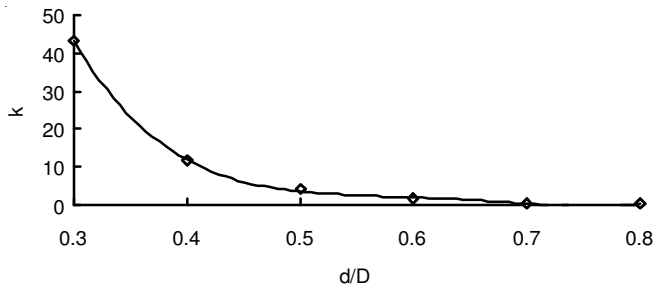


Fig. 8. Relationships between k and d/D for sudden reduction flows

$$k = 1951.5e^{-10.8(d/D)} \tag{9}$$

By fitting the curve in Fig. 8, empirical expression about energy loss coefficient of sudden reduction tube flows can be written as the following:

$$k = 3647.9 \left(\frac{d}{D}\right)^4 - 9204.5 \left(\frac{D}{d}\right)^3 + 8645.3 \left(\frac{d}{D}\right)^2 - 3596.5 \left(\frac{D}{d}\right) + 563 \tag{10}$$

Eqns. 9 and 10 are valid for d/D = 0.4 – 0.8 and R<sub>e</sub> > 10<sup>5</sup>.

**Conclusion**

For sudden enlargement tube flows and sudden reduction tube flows, their energy loss coefficients are all the function of the contraction ratio (d/D) and Reynolds number (R<sub>e</sub>). When Reynolds number R<sub>e</sub> is more than 10<sup>5</sup>, Reynolds number R<sub>e</sub> has little effects on energy loss coefficient. The contraction ratio d/D is the key factor that dominates the energy loss coefficient.

TABLE-1 RELATIONSHIP BETWEEN R <sub>e</sub> AND k						
R <sub>e</sub>	0.1 × 10 <sup>5</sup>	0.4 × 10 <sup>5</sup>	0.8 × 10 <sup>5</sup>	1 × 10 <sup>5</sup>	1.2 × 10 <sup>5</sup>	1.5 × 10 <sup>5</sup>
k (Sudden enlargement flow)	7.89	7.95	8.03	8.11	8.11	8.11
k (Sudden reduction flow)	3.98	3.99	4.09	4.11	4.11	4.11

cient. With the increase of contraction ratio  $d/D$ , energy loss coefficients of sudden enlargement tube flow and sudden reduction tube flow all decrease. By fitting relating curves, energy loss coefficient empirical expressions of sudden enlargement tube flow and sudden reduction tube flow are obtained in this paper.

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