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Experimental Study of a Nonlinear Viscoelastic Model for Elastomeric Bushing in Torsional Mode†

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Elastomeric bushing is a device to cushion the load and moment transmitted from the wheel to the frame of a vehicle in an automotive suspension system. The shape of the elastomeric bushing can be regarded as a hollow cylinder that is bonded to a solid shaft at its inner surface and a hollow sleeve at its outer surface. Because elastomeric bushing shows nonlinear viscoelastic phenomena, the nonlinear viscoelastic model should be used for simulations. In this study, a nonlinear viscoelastic model for the torsional mode of elastomeric bushing was examined and the relationship between the moment and rotational angle in an elastomeric bushing was derived from the experimental data. The nonlinear viscoelastic model for elastomeric bushing in torsional mode was confirmed to be useful by experimental comparisons under the working range.

Key Words: Elastomeric bushing, Viscoelastic model, Torsional mode, Moment relaxation function.

INTRODUCTION

An elastomer is a polymer with a notably low Young's modulus and high yield strain compared to other materials[1]. The term, elastomer, is derived from an elastic polymer and is often used interchangeably with the term rubber. Each of the monomers that link to form a polymer is normally comprised of carbon, hydrogen, oxygen and/or silicon. Elastomers are amorphous polymers existing above their glass transition temperature, so considerable segmental motion is possible. Elastomeric materials exhibit viscoelasticity and are used widely in components, such as tyres and vibration isolators. Elastomeric bushing, which is one type of structural components, isolates vibrations, reduces noise transmission, accommodates oscillatory motions and accepts the misalignment of axes[2]. The shape of an elastomeric bushing can be regarded as a hollow cylinder that is bonded to a solid shaft at its inner surface and a hollow cylindrical sleeve at its outer surface. Fig. 1 shows the configurations of elastomeric bushing. The sleeve is connected to the components of the suspension system and is used to transfer loads and moments from the wheel to the chassis. The elastomeric bushing reduces the shock and vibration in this connection. Dynamics simulations of the automotive suspension system involve interactions between many components. An accurate determination of the loads and moments transmitted between components, motion of the components, stresses in the components and energy dissipation is affected by the quality of the elastomeric bushing model[3].

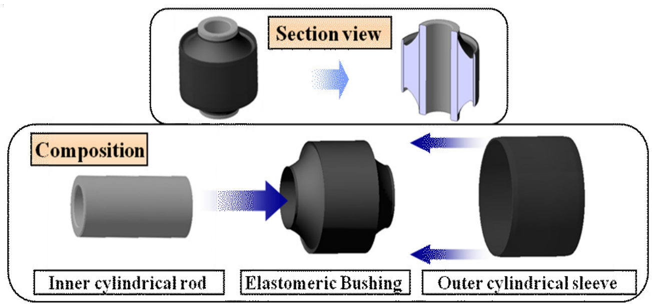

Fig. 1. Configurations of the elastomeric bushing

The classical linear theory of viscoelasticity was first formulated by Boltzman[4]. His work covered the three-dimensional case for isotropic materials. Gurtin and Sternberg[5] suggested a mathematical representation of the constitutive equation for linear viscoelastic materials and developed equations for boundary and stress analysis problems. The developments of computational ability allow researchers to carry out numerical

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simulations[6-8]. Green and Rivlin introduced and developed a multiple integral representation for nonlinear viscoelasticity to describe the real behaviour of commonly used materials more accurately[9]. Coleman and Noll developed a general three-dimensional constitutive equation for an isotropic viscoelastic solid with fading memory[10]. Pipkin-Rogers[11] introduced a constitutive equation for nonlinear viscoelastic response of polymers. Using a modified superposition concept, they constructed a single integral model based on single step relaxation data. They outlined a procedure for improving the accuracy of the model by including multi-step relaxation data and also extended this model to a full three-dimensional setting. The integrand in the three-dimensional Coleman-Noll model depends on both the current value of the finite strain tensor and the history of the relative stretch tensor and contains a large number of terms. In contrast, the integrand in the Pipkin-Rogers model depends only on the history of a finite strain tensor. Step rotational tests are unsuitable for determining the integrand of the Coleman-Noll model, whereas these tests can be used to determine the integrand of the Pipkin-Rogers model. The Pipkin-Rogers modeling concept can also be extended readily to the load-deformation response.

Elastomeric bushing exhibits complicated relationships among the applied loads, geometry of deformation, time and other factors. Deformations and rotations about several axes are related to their corresponding loads and moments. The elastomeric material causes a nonlinear and time dependent relationship between the corresponding load and deformation[3]. One-dimensional tests on bushings, in radial or axial motion, were carried out at the Center for Automotive Structural Durability Simulation in the Department of Mechanical Engineering and Applied Mechanics at the University of Michigan[12]. The experimental data from these one-dimensional tests in radial mode suggest that the force depends on the displacement in a nonlinear manner and that the nonlinearity is manifested under normal operating conditions. The data also suggests that the elastomeric bushings exhibit the features of a viscoelastic response. An accurate representation of the nonlinear viscoelastic response is important for an accurate dynamics simulation.

In this study, torsional mode was used to obtain the relationship between the moment and rotational angle through experimental research. This study examined an elastomeric bushing model in torsional mode and compared the proposed moment-rotational angle relationship with the experimental results. Because the test could not control temperature, the temperature effects were not considered. Moreover, although factors, such as aging and frequency of moment, affect the conditioning of the elastomeric bushing, they were not considered in this research. In addition, it was assumed that the microstructure of the elastomeric bushing would be fairly constant during testing. A future study will include these parameters.

The next section introduces the proposed viscoelastic relationship between the moment and rotational angle and outlines the method for determining the moment relaxation property. The experimental results are presented and the relaxation property is determined. Finally we report the predicted quality and conclusions.

Proposed viscoelastic relation between the moment and the rotational angle: Pipkin and Rogers[11] introduced the simplest relationship between the moment and rotational angle for the nonlinear viscoelastic response of polymers. Applying integration by parts to the simple relation of Pipkin and Rogers produced the following form.

$$M(t) = R(\phi(0), t) + \int_0^t \frac{\partial R(\phi(s), t-s)}{\partial \phi(s)} \frac{d\phi(s)}{ds} ds \qquad (1)$$

where M(t) is the moment at current time t, s is the time, ϕ is the rotational angle and $R(\phi,s)$ is the rotational angle dependent moment relaxation function of time s. In particular, $R(\phi_0, s)$ is a property of the elastomeric bushing material and represents the moment at time s for a step rotational angle, ϕ_0. This is reason that $R(\phi, s)$ is called the rotational angle dependent moment relaxation function at time s. When s increases, $R(\phi, s)$ decreases monotonically and $R(0^-, s) = 0$ for $s < 0$. Ideally, $R(\phi, s)$ can be determined where the inner rod undergoes a step rotational angle with respect to the outer sleeve. However, this ideal process cannot be realized due to the inertia of the testing apparatus. To solve these experimental problems, the ramp to constant rotational angle history was considered instead of the step rotational angle history. Therefore, the rotational angle control test should be performed with the ramp to a constant rotational angle history. Accordingly, the rotational angle dependent moment relaxation functions can be obtained using the moment extrapolation method[3].

EXPERIMENTAL

In this study, a single mode, particularly torsional mode, was considered in the elastomeric bushing material. The elastomeric bushing was fixed at its outer radius and the inner rod was subjected to a rotational angle $\phi(s)$. The ramp to constant rotational angle histories can be expressed as follows:

$$\begin{aligned} \phi(s) &= \frac{\phi_i}{T_j^*} t, \qquad 0 \le s \le T_j^* \\ &= \phi_i, \qquad T_j^* \le s \le 120\,\text{sec}. \end{aligned} \qquad (2)$$

$$i = 1, 2, 3, 4, 5, \qquad j = 1, 2, 3, 4.$$

The slope changed from positive to zero at T^*_j, which is called the rise time.

In this study, the rise times are given as $T^*_1 = 1$ sec, $T^*_2 = 2$ sec., $T^*_3 = 4$ sec., $T^*_4 = 8$ sec. and the constant rotational angles are $\phi_1 = 2°$, $\phi_2 = 4°$, $\phi_3 = 6°$, $\phi_8 = 8°$, $\phi_5 = 10°$. Consequently, there are twenty data sets with the given rise times ($T^*_1 = 1$ sec, $T^*_2 = 2$ sec., $T^*_3 = 4$ sec., $T^*_4 = 8$ sec) and rotational angles ($\phi_1 = 2°$, $\phi_2 = 4°$, $\phi_3 = 6°$, $\phi_8 = 8°$, $\phi_5 = 10°$).

The ideal type of test is a step rotational angle control test. On the other hand, a true step rotational angle control test is not possible due to inertia of the testing equipment. Therefore, a ramp to constant rotational angle was used instead of a step rotational angle. The test controller was programmed to increase the rotational angle at a constant rate during the period from time zero to a rise time T^* and hold it at a fixed ϕ_i after the rise time T^*. As the rise time T^* decreases, the ramp to the constant rotational angle control test approaches to the step rotational angle control test. As shown in eqn. (2), 20 ramp to constant rotational angle control tests were carried out until t = 2 min. For each ramp to constant rotational angle control tests, the peak moment occurred when the prescribed rotational angle changed from increasing to being held constant and the peak moment was the greatest at the shortest rise time, T* = 1 sec. The moment was relaxed to equilibrium during the period in which the rotational angle remained constant. For reference, Fig. 2 shows the moment outputs of the rotational angle control tests for $\phi_i = 4^o, 8^o, T^*_j = 1, 2, 4, 8$ sec..

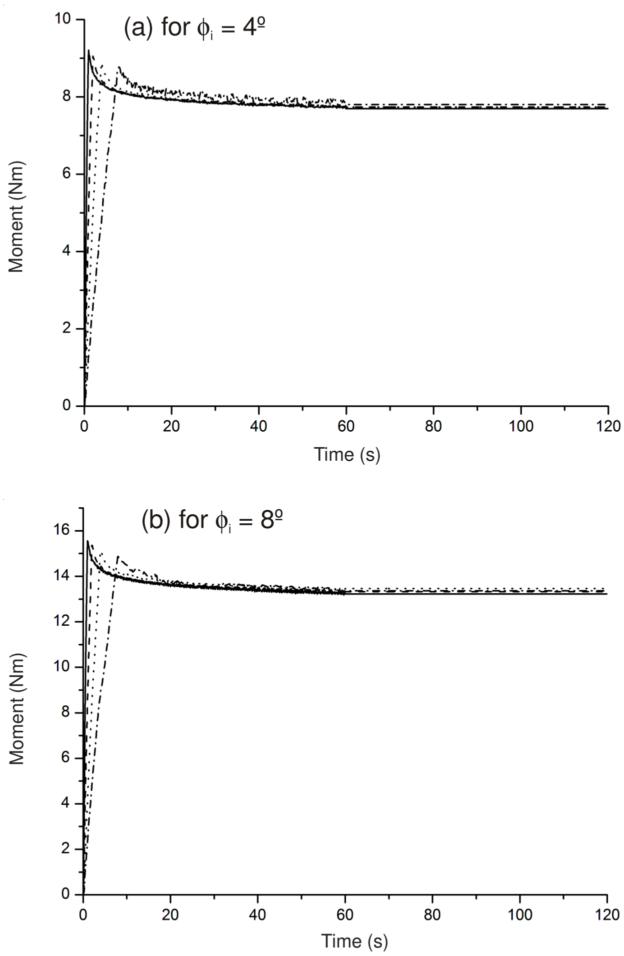

Fig. 2. Experimental moment outputs for $f_i = 4, 8^o, T^* = 1, 2, 3, 4$ sec

Mathematical representations: Using the moment extrapolation method in the previous section for $\phi_1 = 2^o, \phi_2 = 4^o, \phi_3 = 6^o, \phi_8 = 8^o, \phi_5 = 10^o, T^*_1 = 1$ sec, $T^*_2 = 2$ sec., $T^*_3 = 4$ sec., $T^*_4 = 8$ sec., the extrapolation process was carried out and the values of the moment relaxation functions for $0 \le s \le 60$ sec. were obtained. Because the moment is odd in the rotational angle, ϕ, the rotational angle dependent moment relaxation function, R(ϕ, s), contains only odd powers of the rotational angle, ϕ. The time coefficients of ϕ and ϕ^3 are sufficient for the representations because the time coefficients over ϕ^5 are approximately zero. Therefore, R(ϕ, s) can be expressed as follows:

$$R(\phi, s) = \phi G_1(s) + \phi^3 G_3(s). \tag{3}$$

The rotational angle dependent moment relaxation function provides a data set that can be used to determine the time coefficients $G_1(t_a)$ and $G_3(t_a)$. If $\phi^{(\gamma)}$ is the value of a step rotational angle data set for $\gamma = 1,3$, $R(\phi^{(1)}, t_a)$ and $R(\phi^{(3)}, t_a)$ can be expressed as follows:

$$R(\phi^{(1)}, t_a) = \phi^{(1)} G_1(t_a) + \{\phi^{(1)}\}^3 G_3(t_a), \quad R(\phi^{(3)}, t_a) = \phi^{(3)} G_1(t_a) + \{\phi^{(3)}\}^3 G_3(t_a), \tag{4}$$

where a = 1,2,3,...., 601, $t_1 = 0$ sec., $t_2 = 0.1$ sec., $t_3 = 0.2$ sec., $t_4 = 0.3$ sec., ..., $t_{601} = 60$ sec.

The coefficients, $G_1(t_a)$ and $G_3(t_a)$ were obtained at a set of times t_a for a = 1,2,3,...., 601, $t_1 = 0$ sec., $t_2 = 0.1$ sec., $t_3 = 0.2$ sec., $t_4 = 0.3$ sec., ..., $t_{601} = 60$ sec. from a minimization of the least-squares error.

This discrete data set $G_1(t_a)$ and $G_3(t_a)$ can be represented as a sum of exponential functions, as follows:

$$G_1(t) = C_{11} + C_{12} e^{-t/\tau_{12}} + C_{13} e^{-t/\tau_{13}}, \quad G_3(t) = C_{31} + C_{32} e^{-t/\tau_{32}} + C_{33} e^{-t/\tau_{33}} \tag{5}$$

The parameters $(C_{i\alpha}, \tau_{i\beta}, \alpha = 1,2,3, \beta = 2,3)$ were obtained using the nonlinear least-squares method. After applying a 3 % relative error to the fitting process, the time coefficient function $G_1(t)$ and $G_3(t)$ obtained can be rewritten as follows:

$$G_1(t) = 2.28 + 0.16 e^{-t/17} + 0.31 e^{-t/2}, \quad G_3(t) = -0.018 - 0.004 e^{-t/2} \tag{6}$$

Finally, the complete form of the nonlinear viscoelastic model for the elastomeric bushing in torsional mode was obtained as follows:

$$M(t) = R(\phi(0), t) + \int_0^t \left\{ \frac{\partial R(\phi(s), t-s)}{\partial \phi(s)} \frac{d}{ds}(\phi(s)) \right\} ds, \tag{7}$$

where $R(\phi(s), t) = [2.28 + 0.16e^{-t/17} + 0.31e^{-t/2}]\phi(s) + [-0.018 - 0.004e^{-t/2}](\phi(s))^3$.

RESULTS AND DISCUSSION

Predictive quality of the proposed model: Here we discuss the predictive capability of the proposed model. The moment outputs for the ramp to constant rotational angle histories are considered for the proposed model and experiment. Both the experimental results and the results of the proposed model were used to determine the moment responses to the specified rotational histories for $0 \le t \le 60$ sec. For evaluation purposes, the relative error E was defined using the 2-norm concept and is expressed as follows:

$$E = \frac{\|(\text{the proposed model outputs}) - (\text{Experimental results})\|_2}{\|(\text{Experimental results})\|_2} \times 100\,(\%) \tag{8}$$

The particular rotational angle histories for the experiment and proposed model can be expressed as follows:

$$\phi(s) = \frac{10^o}{T_j^*} s, \quad 0 \le s \le T_j^* \\ = 10^o, \quad T_j^* \le s \le 60 \text{ sec.} \\ T_j^* = 1, 2, 4, 8 \text{ sec.} \tag{9}$$

A comparison of the proposed model outputs with the experimental results was carried out for rise times, $T^* = 1, 2, 4, 8$ sec. For $\phi = 10^o$ and $T^* = 1, 2, 4, 8$ sec., the relative errors between the results of the proposed model and the experimental results were less than 5 %. Fig. 3 shows the comparisons between the proposed model outputs and the experimental results for $\phi = 10^o$ and $T^* = 1, 4$ sec. In Fig. 3, the solid line denotes the experimental results and the dotted line shows the results of the proposed model. The proposed model appears to show possible agreement with the experimental results under practical operating regions.

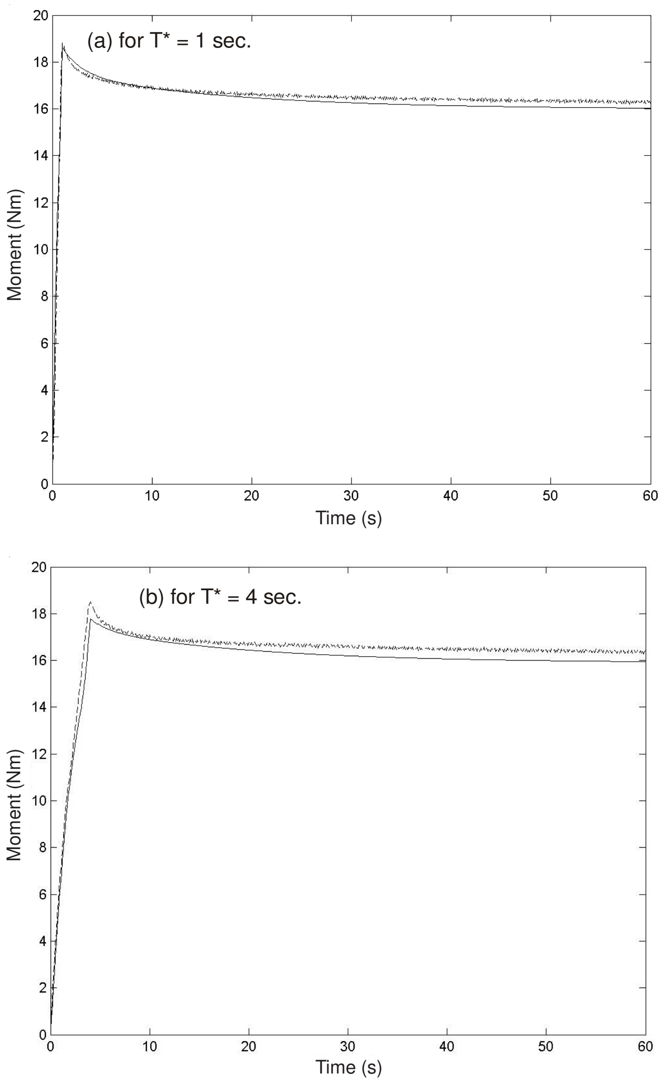

Fig. 3. Comparisons between the proposed model outputs and experimental results for $\phi = 10°$ and $T^* = 1, 4$ sec

Conclusion

The moment-rotational angle relationship for torsional mode of an elastomeric bushing was studied experimentally. The proposed approximate relationship was expressed in terms of the moment relaxation property determined from the experimental results. The moment relaxation property in the explicit moment-rotational angle relation was determined using a method that extrapolates the results obtained experimentally. Because the comparisons were carried out for only a limited number of rotational angle histories, the results were only satisfactory over limited times and deformation ranges. Nevertheless, this can be acceptable for use in multi-body dynamics simulations involving short time intervals.

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