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# Effect of Dimensionless Hydrodynamic Parameters on Coarse Particles Flotation

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The upper particle size is an important parameter in flotation operation and the sharp decrease of flotation response when particle size approaches such a limit is well known phenomenon. Nowadays, particle size is often measured and controlled in flotation concentrators. In this study the dependence of the dimensionless hydrodynamic parameters (Reynolds Number-Re, Froude Number-Fr and Weber Number-We) on the particle size variation was investigated on the quartz particles using laboratory mechanical flotation cell. Maximum flotation response was observed at Re = 89800, Fr = 2.4 and We = 1558. For either more quiescent (Re < 73500 and Fr < 1.61) or more turbulent (Re > 106200 and Fr > 3.35) conditions, flotation recovery decreased steadily. Under more quiescent hydrodynamic conditions, the lower recovery was due to the fact that the impeller was not capable of keeping particles in suspension properly whereas under more turbulent conditions the disruption of particle/bubble aggregate was intensively observed. Furthermore, amount of collision probabilities is calculated using various equations. According to this study, with increasing particle size or impeller velocity, increase in probability of collision and with increasing air flow rate, decrease in probability of collision was observed.

**Key Words: Flotation, Coarse particle, Dimensionless parameters, Collision probability, Quartz.**

## INTRODUCTION

The primary aim in mechanical flotation cells is the selective attachment of hydrophobic particles to air bubbles under dynamic conditions that are generated by the action of an impeller. Thus, it is useful to consider the extent to which hydrodynamic parameters influence flotation performance since they play a major role in particle/bubble collision, attachment

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and transport within an environment that holds some degree of turbulence[1-3]. Froth flotation is effective only in a relatively narrow particle size range, approximately in the range of 10 to 100 μ in diameter[4,5].

Hydrodynamic analyses of bubble-particle interactions conducted by many investigators suggest that the difficulty in floating fine particles is due to the low probability of bubble-particle collision, while the problem with coarse particles can be attributed to the high probability of detachment[6]. In present studies, the influence of some dimensionless hydrodynamic parameters (Reynolds Number-Re, Froude Number-Fr and Weber Number-We) on flotation performance of particles of quartz has been carried out. Furthermore, probability of particle-bubble collision can be calculated using intermediate first and second and potential equations.

## EXPERIMENTAL

Quartz particles (specific gravity = 2.65 g/cm$^3$) of four diameter classes were used contain: -300+212, -420+300, -500+420 and -590+500 μ. The collector used in the flotation tests was oleic acid (1000 g/ton) at pH = 12.5 and the frother used in the flotation tests was MIBC (75 g/ton). Sodium hydroxide (analytical grade) was used for pH regulation. Anionic flotation of quartz in pH = 12.5 attribute to $Ca^{2+}$ present and activation of quartz surface with this $OH^-$.

Flotation tests were carried out in a mechanical cell. An impeller diameter of 0.07 m was used for pulp agitation and a cell with square section was used that its length and height were 0.12 and 0.1 meters, respectively. Impeller rotating speed was 700, 900, 1100 and 1300 rpm and air flow rate was 15, 30, 45 and 75 L/h. Pulp viscosity was calculated by eqn. 1 proposed by Roscoe[7]:

$$\mu_p = \mu_w(1-\phi)^{-2.5} \quad (1)$$

in which, $\mu_p$ = pulp viscosity, $\mu_w$ = water viscosity and $\phi$ = fraction of pulp volume occupied by solids.

Reynolds number-Re is ratio between inertial and viscous forces, Froude Number-Fr is ratio between inertial and gravity forces and Weber Number-We is ratio between inertial and capillary forces. So, these dimensionless hydrodynamic parameters were calculated[8] using eqns. 2-4:

$$\mathrm{Re} = \left(ND^2\rho\right)/\mu_p \quad (2)$$

$$\mathrm{Fr} = \left(N^2 D\right)/g \quad (3)$$

$$\mathrm{We} = \left(N^2 D^3 \rho\right)/\gamma \quad (4)$$

in which, $\rho$ = pulp density, $\mu_p$ = pulp dynamic viscosity, g = gravity acceleration, $\gamma$ = surface tension of air/solution interface, N = impeller speed and D = impeller diameter.

## RESULTS AND DISCUSSION

### Variation of dimensionless hydrodynamic parameters and air flow rates with flotation response of quartz particles

For different air flow rate, the influence of Reynolds number, Froude number and Weber number on flotation response of quartz particles in four classes are illustrated in Fig. 1. Quartz particles presented a plateau of maximum recovery at Re = 89800, Fr = 2.4 and We = 1558. For either more quiescent (Re < 73500 and Fr < 1.61) or more turbulent (Re > 106200 and Fr > 3.35) conditions, flotation recovery decreased steadily. This behaviour suggests that the flotability demands some turbulence to promote particle-bubble collision but that turbulence may not be high enough to destroy particle-bubble aggregates. A good review on the effect of turbulence on particle/bubble attachment and detachment is reported elsewhere[1-3].

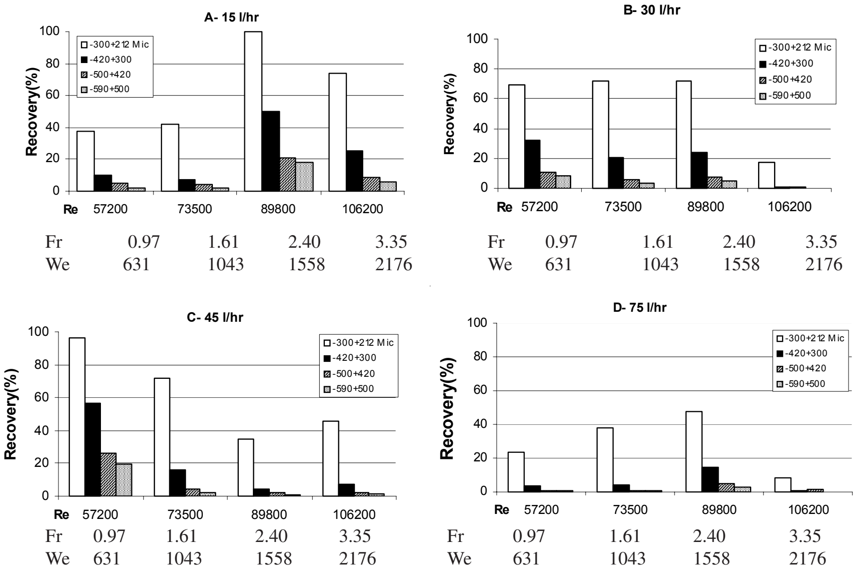

Fig. 1. Flotation response of quartz particles *vs.* dimensionless hydrodynamic parameters (Re, Fr and We) at various air flow rate

In this study, a range of 57200 < Re < 106200 was explored in flotation tests when Impeller speeds changes from 700 to 900 and 1100 to 1300 rpm. According to Fig. 1, coarse quartz particles showed a pronounced lower recovery than finer ones. It seems that bigger particles demand much more turbulence to become suspended and collide with air bubbles than smaller ones.

According to Fig. 1 (Air flow rate = 15 L/h), under the dominance of inertial forces ($Fr > 3.35$) and/or turbulent flow ($Re > 106200$), the flotation response of quartz particles decreased sharply to values close to zero. The same behaviour was also observed for different air flow rates (30, 45 and 75 L/h). The results suggest that, regardless of particle size and air flow rate, the aggregates of particle/bubble formed in the pulp, after successful collision and attachment, are likely to be destroyed under the dominance of the severe hydrodynamic conditions characterized by $Re > 106200$ and $Fr > 3.35$.

A range of $57200 < Re < 106200$ was explored in flotation tests carried out with *vs.* particle sizes and air flow rates. A plateau of maximum recovery was observed at $We = 1558$ and for $We > 2176$, no froth layer was observed during flotation experiments and under such a special condition, the recovery was nil. In this research, flotation tests were carried out in four different flow rates from 15 to 30 and 45 to 75 L/h. The results suggest that with increasing air flow rate, flotation recovery decrease to such a level that when air flow rate is 75 L/h recovery was found to be minimum.

**Bubble size distribution and raise velocity:** Although flotation has been used as a process for the removal of suspended particles from mineral slurries and wastewater for many years, the characterization of the bubbles used and their technical role has attracted interest recently. The flotation process depends on the ability of bubbles to collect particles from the suspension and carry them to the surface where a layer of froth or sludge can be removed over a launder. In ore processing the particles to be floated are first made hydrophobic. The collection mechanism is one in which the hydrophobic particles attach to bubbles (> 600 μm in diameter) by the formation of a finite contact angle at the gas-liquid-solid contact line[9] .

In present studies, bubble size distribution raise velocity was measured similar to McGill bubble viewer. It consists of a sampling tube attached to a viewing chamber with a window inclined 15° from vertical. The closed assembly is filled with water of similar nature to that in the flotation cell (to limit changes in bubble environment during sampling) and the tube is immersed to the desired location below the froth. Bubble raise into the viewing chamber and are imaged by a digital video camera as they slide up the inclined window illuminated from behind[10]. The mean bubble diameter adopted was the Sauter diameter, calculated by the equation[9]:

$$d_{32} = \frac{\sum n_i d_i^3}{\sum n_i d_i^2} \tag{5}$$

in which n, is number of bubbles and d, is bubble diameter. The effect of impeller speed on Sauter mean bubble diameter at different air flow rate has shown in Fig. 2 and Table-1. The bubble size decrease with impeller speed increase, more evident at the low air flow rate. The effect of air flow rate is shown in Fig. 2, as can be seen, bubble size increased with increasing air flow rate.

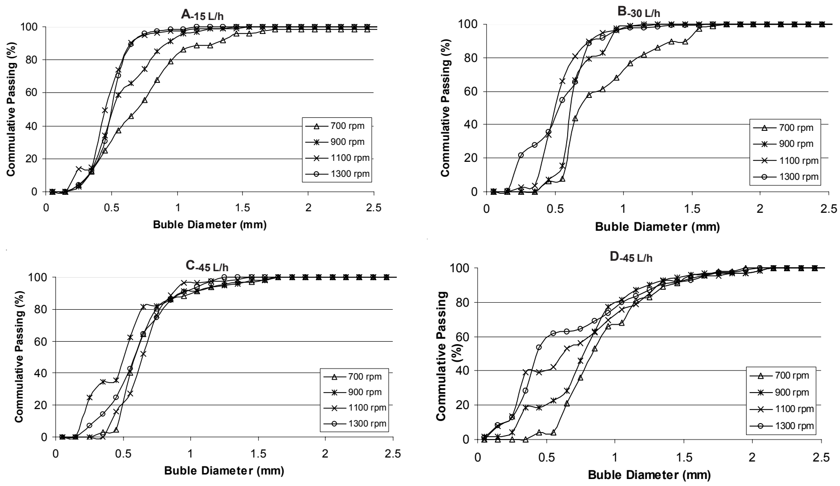

Fig. 2. Effect of the impeller speed on bubble size distribution at different air flow rate

TABLE-1
BUBBLE SIZE DISTRIBUTION, RAISE VELOCITY AND REYNOLDS NUMBER

| Air flow rate (L/h) | Impeller speed (rpm) | Bubble diameter (mm) | Bubble raise velocity (cm/s) | Bubble Reynolds numbers |
|---|---|---|---|---|
| 15 | 700 | 1.34 | 16.58 | 248 |
| | 900 | 0.83 | 16.40 | 135 |
| | 1100 | 0.65 | 14.10 | 102 |
| | 1300 | 0.55 | 16.26 | 100 |
| 30 | 700 | 1.02 | 16.68 | 190 |
| | 900 | 0.68 | 17.69 | 134 |
| | 1100 | 0.69 | 17.90 | 138 |
| | 1300 | 0.83 | 18.47 | 171 |
| 45 | 700 | 0.96 | 16.73 | 179 |
| | 900 | 0.95 | 17.58 | 186 |
| | 1100 | 0.71 | 17.86 | 141 |
| | 1300 | 0.82 | 18.96 | 174 |
| 75 | 700 | 1.26 | 18.02 | 253 |
| | 900 | 1.21 | 18.21 | 246 |
| | 1100 | 1.40 | 18.45 | 288 |
| | 1300 | 1.52 | 19.28 | 327 |

Influence parameters in bubble Reynolds number are bubble raise velocity, bubble diameter and density and dynamic viscosity of fluid around of bubble. Bubble Reynolds number is calculated from below equation[3]:

$$\mathrm{Re}_b = V_b d_b \rho_f / \eta \qquad (6)$$

in which, $V_b$ is bubble raise velocity, $d_b$ is bubble diameter, $\eta$ is fluid dynamic viscosity and $\rho_f$ is fluid density. According to Table-1 and Fig. 3, when air flow rate is 15 L/h, with increasing impeller speed, the bubble raise changes from 16.58 to 14.6, 14.1 and 16.28 cm/s, respectively, but if air flow rate increase to 30, 45 and 75 L/h increasing impeller velocity cause to increase bubble raise velocity.

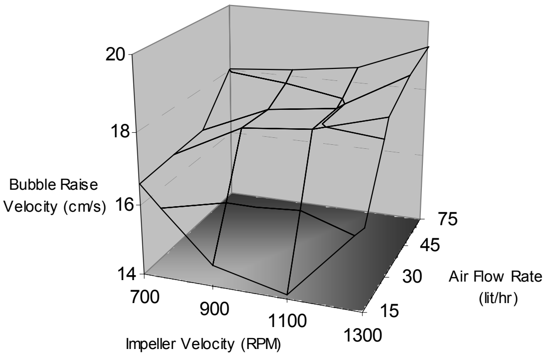

Fig. 3. Effect of the impeller speed on bubble raise velocity at different air flow rates

Minimum bubble raise velocity is 14.1 cm/s around 15 L/h air flow rate and 1100 rpm impeller velocity and maximum bubble raise velocity is 19.28 cm/s around 75 L/h air flow rate and 1300 rpm impeller velocity. In these tests, bubble Reynolds numbers was changed between 100 to 327.

**Collision probability:** The probability (P) of a particle being collected by an air bubble in the pulp phase of a flotation cell[6] can be given by:

$$P = P_c P_a \left(1 - P_d\right) \qquad (7)$$

in which $P_c$ is the probability of bubble particle collision, $P_a$ is the probability of adhesion and $P_d$ is the probability of detachment. There is a generalized equation[6] for calculation $P_c$ as given below:

$$P_c = A\left(\frac{d_p}{d_b}\right)^n \qquad (8)$$

in which $d_p$ is the diameter of particle, $d_b$ is the diameter of bubble and A and n are the parameters that vary with Reynolds numbers. Table-2 gives these values for the three different flow regimes considered, *i.e.*, Stokes, intermediate and potential flows.

TABLE-2
VALUES OF A AND N FOR DIFFERENT FLOW CONDITIONS[6]

| Flow conditions | A | n |
|---|---|---|
| Stokes (Ref. 11) | 2/3 | 2 |
| Intermediate I (Ref. 12) | $\left[\frac{3}{2}+\frac{4\,\mathrm{Re}^{0.72}}{15}\right]$ | 2 |
| Intermediate II (Ref. 13) | $\frac{3}{1}\left[1+\frac{(3/16)\mathrm{Re}}{1+0.249\,\mathrm{Re}^{0.56}}\right]$ | 2 |
| Potential (Ref. 14) | 3 | 1 |

Probability of collision was calculated for all of flotation tests using different equations in Table-1. When collision probability calculate using Intermediate I[12], Intermediate II[13] and Potential equation[14], amount of collision probabilities are exaggerated but Stokes equation[11] can estimate probability of collision. According to Fig. 4, as the particle size increase, the probability of collision increase and similarly with increasing air flow rates the probability of collision increase.

## Conclusion

Flotation response of quartz particles of four classes (-300+212, -420+300, -500+420 and -59+500 with oleic acid (1000 g/ton), at pH = 12.5, was highly influenced by hydrodynamic conditions under which the tests were performed. A plateau of maximum recovery was observed at Re = 89800, Fr = 2.40 and We = 1558. For either more quiescent (Re < 73500 and Fr < 1.61) or more turbulent (Re > 106200 and Fr > 3.35) conditions, flotation recovery decreased steadily. By visual observation, it was possible to see that under more quiescent hydrodynamic conditions (Re < 73500 and Fr < 2.4), the impeller was not capable of keeping particles in suspension and promoting particle/bubble collision on the other hand, under more turbulent conditions (Re < 106200 and Fr < 3.35), the disruption of particle/bubble aggregates was easily observed. Probability of collision is calculated for flotation tests using different equations in Table-1. When collision probability calculate using Intermediate I, Intermediate II and Potential equation, amount of collision probabilities are exaggerated but Stokes equation can estimate probability of collision. According to Fig. 4,

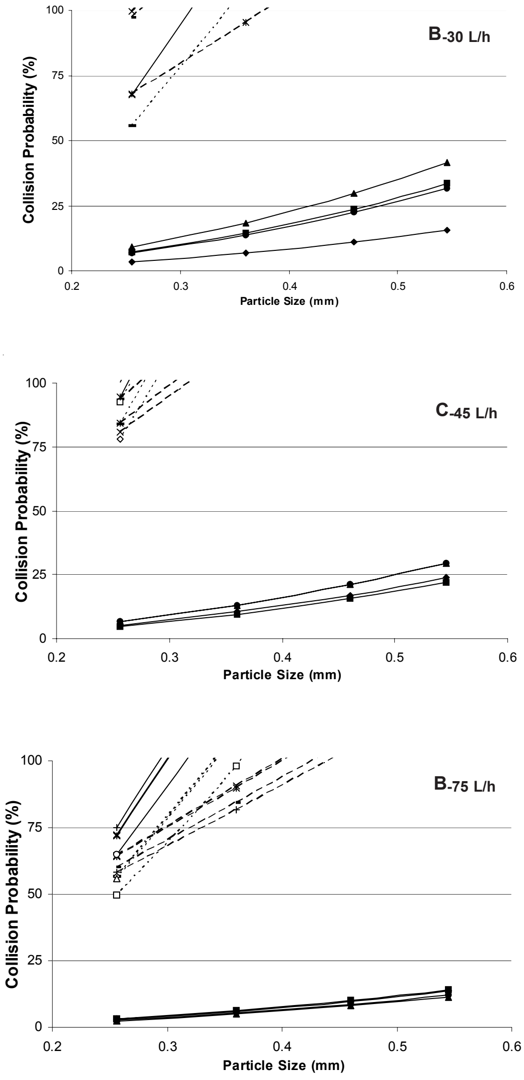

Fig. 4. Effect of the impeller speed on probability of collision at different air flow rates

as the particle size increase, the probability of collision increase and similarly with increasing air flow rates the probability of collision increase.

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