

## Mathematical Modelling of Soil Hydraulic Properties and Numerical Analyses of Moisture Dynamic

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For determination of the state of soil water, which is a function of time and depth in soil system, water flux in soil should be modelled. Therefore, designating a suitable model yielding the interactions between soil moisture content (W), water potential (P) and hydraulic conductivity (K) is the prerequisite. Some samples collected from the some physical and chemical properties. Hydraulic conductivity and soil moisture characteristics of the studied soil were determined by utilizing SIMONA (SIMONA, is basic model describing physical and bio physical processes in agro ecosystem) software most suitable model known at present. For the solution of soil moisture dynamic equation finite differences method was used. The soil water dynamics values obtained from the model and laboratory measurements were also correlated in the studies.

**Key Words: Mathematical modelling, Soil moisture.**

### INTRODUCTION

Soil is a very complex system due to the numerous individual and mutual factors affecting irrigation, soil amelioration, illuvation, evapotranspiration and on circulation of pollutant materials applied to soil. Thus, determination of quantity and direction of water flow is very important<sup>1</sup> for sustainable land management.

Plant development models are being widely used in recent years. Approaches for developing models may be different. The basic principle of models is generally based on block system. In all plant development models, soil water flow block is always used<sup>2-4</sup>. Thus, one of the important

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dynamic block is soil flux and for modelling of time and depth dependent process. The two basic soil hydro physical properties, namely soil moisture dynamic and hydraulic conductivity should be well defined for accurate determination.

Soil suction curve and hydraulic conductivity should be determined initially for utilizing the model. Any changes of moisture content of certain layers at certain periods are determined by calculating water input and output of the layer. Energy and mass transfer in soils are expressed by diffusion processes. Thus diffusion formulas used in mathematics and physics are used for determination of these transfer. For adjusting heat flux, observation of soil moisture content, soil hydraulic conductivity and differential moisture content capacity are in accordance with above mentioned diffusion processes.

Any change of soil moisture content ( $\Delta w$ ) at  $\Delta t = t_{k+1} - t_k$  time increment is equal to water input and output of a layer and expressed as below;

$$\frac{\Delta W}{\Delta t} = -\frac{\Delta V}{\Delta x} \quad (1)$$

Water flux level in soil is calculated according to Darcy law,

$$V = -K \left( \frac{\Delta P}{\Delta x} - 1 \right) \quad (2)$$

V: water flux in soil

Change in moisture content at  $(t_{k+1} - t_k)$  time is  $h_j$ ; F is the thickness of the layer, area;

$$[W(t_{k+1}) - W(t_k)] h_j F = (V_{j+1} - V_j) F \Delta t \quad (3)$$

$(V_{j+1} - V_j)$ : change of flux in the layer when flux is measured in layers  $j-1, j$  and  $j+1$  according to Darcy law in eqn. 3 then equation below is attained

$$V_{j+1} = -K_{j+1} \left( \frac{P_{+1j} - P_j}{h_j} - 1 \right) \quad (4)$$

where,  $h_j = \text{constant}$

when,  $K_{j+1} = K_j = K_{j-1} = K$ , is accepted as constant and  $(V_j$  and  $V_{j+1})$  flux, mentioned above is taken as:

$$V_j = -K_{j+1} \left( \frac{P_j - P_{j-1}}{h_j} - 1 \right) \quad (5)$$

Then, the final equation is:

$$[W_j(t_{k+1}) - W_j(t_k)] h_j = \left( K \left( \frac{P_{j+1} - P_j}{h_j} - 1 \right) \right) - \left( K \left( \frac{P_j - P_{j-1}}{h_j} - 1 \right) \right) \Delta t \quad (6)$$

If equation is simplified in expression as: ( $h = h_1 = h_2 \dots = h_j$ )

$$[W_j(t_{k+1}) - W_j(t_k)] = \frac{K\Delta t}{h^2} (P_{j+1} - 2P_j + P_{j-1}) \quad (7)$$

If,  $\frac{K\Delta t}{h^2} = A_t$  is symbolized as  $A_t$  ( $1 \text{ cm}^{-1}$ ).

$$[W_j(t_{k+1}) - W_j(t_k)] = A_t (P_{j+1} - 2P_j + P_{j-1}) \quad (8)$$

For  $W_j$  layer the final equation is

$$W_j(t_{k+1}) = W_j(t_k) + A_t (P_{j+1} - 2P_j + P_{j-1}) \quad (9)$$

The value of  $P$  is (cm and therefore  $W_j(t_{k+1})$  is dimensionless. Moreover, the conditions of upper and lower limits (at surface  $v = 0$ ) can be expressed as follows at selected NR depth,  $X = XR$

$$\begin{aligned} -K|(\partial P)(\partial X)| &= E_s - R \\ X &= 0 \end{aligned} \quad (10)$$

$$\begin{aligned} -K|(P)(\partial P/\partial X)| &= 0 \\ X &= NR \end{aligned} \quad (11)$$

Evaporated water and precipitation levels are expressed as  $E_s$  and  $R$  respectively in the formula.

Water moisture curve and hydraulic conductivity used in the formula are expressed as:

$$P = P_d \exp[A^*(W_d - W)] \quad (12)$$

$$K = K_f (P_d/P)^m \quad (13)$$

Soil water potential and saturated soil moisture contents are expressed as  $P_d$  and  $W_d$ , respectively. In the equation,  $A$  and  $m$  are semiempirical coefficients. Soil evaporation, which is an effective factor on soil water flux, model is:

$$E_s = \rho_a * D_s (q_s(0) - q_a) \quad (14)$$

where,  $\rho_a$ : density of air ( $\text{g cm}^{-3}$ ),  $D_s$ : molecular moisture conductivity ( $\text{cm s}^{-1}$ ) between soil and atmosphere, to wind velocity,  $q_s(0)$  and  $q_a$ : are the soil surface and atmosphere's absolute moisture contents, respectively ( $\text{g H}_2\text{O g}^{-1}$  air). Generally absolute moisture content is a function of temperature ( $T$ ) according to Magnus law<sup>5</sup>

$$q(T) = 3.79 * 10^{-3} \exp(17.1 * T / (235 + T)) \quad (15)$$

Initial condition of model is:

$$f(W_i) = W_i \quad (16)$$

Mathematical model of soil water flux should be analytical in relation to depth at some periods of other blocks.

## EXPERIMENTAL

The soil sample used in the study was taken from the Experimental and Research Station of the Department of Soil Science of Agricultural Faculty of University of Çukurova, Turkey. Dry bulk density was measured by the core method<sup>6</sup>, saturated hydraulic conductivity was determined according to the method<sup>7</sup> and particle size distribution was determined by the Bouyoucos hydrometer method<sup>8</sup>. pH and total salt were all determined according to the reported method<sup>9</sup>. Water retention capacity (pF) was measured according to the known method<sup>10</sup>. Some properties of soil are given in Tables 1 and 2.

TABLE-1  
SOME PHYSICAL AND CHEMICAL PROPERTIES OF THE SOIL

Properties	Depth (cm)				
	0-20	20-40	40-60	60-80	80-100
pH	7.960	7.980	8.100	8.11	8.22
Salinity (%)	0.065	0.056	0.054	0.05	0.04
Bulk density (g cm <sup>-3</sup> )	1.290	1.560	1.600	1.67	1.68
Hydraulic conductivity (cm h <sup>-1</sup> )	3.190	1.710	1.930	1.52	0.67
Sand (%)	21.920	17.730	16.200	22.63	26.43
Silt (%)	37.990	41.130	41.570	40.86	40.06
Clay (%)	40.090	41.140	42.230	36.51	33.51

TABLE-2  
SOIL WATER CHARACTERISTIC VALUES

Pressure (bar)	Depth (cm)				
	0-20	20-40	40-60	60-80	80-100
0.010	52.23	50.86	44.08	41.17	41.78
0.100	42.58	39.20	35.58	34.15	33.41
0.333	40.06	36.60	33.22	31.72	31.13
1.000	38.36	34.46	30.94	19.23	29.86
5.000	18.06	18.20	17.49	15.68	14.63
15.000	17.09	17.16	16.89	15.11	13.94

## RESULTS AND DISCUSSION

Soil water flux model, defined by eqns. 1-14, was used for simulating soil moisture contents of each 10 cm layers of 100 cm soil profiles, which is related to time, by utilizing visual programming for Delphi 3.0 (Computer Program Language) software<sup>11</sup>. By using the data in Tables 1 and 2, the eqns. 12 and 13 were used with the application of pF model bank in the

simulation model. Eqns. 12 and 13 used in the model  $P_d = -5$  cm,  $W_d = 0.48$   $\text{cm}^3 \text{cm}^{-3}$ ,  $A = 24$ ,  $K_f = 0.87$   $\text{cm h}^{-1}$ ,  $m = 2.1$ ,  $\rho_a = 1.2 \times 10^{-3}$   $\text{g cm}^{-3}$ ,  $D_s = 1$   $\text{cm s}^{-1}$  values are used for identification. Besides, for the certain block the initial moisture contents of the input parameters below are used (Table-3).

TABLE-3

Depth (cm)	Initial soil water content ( $\text{cm}^3 \text{cm}^{-3}$ )	Initial soil water potential (cm H <sub>2</sub> O)
0-10	$W(0) = 0.17$	$P(0) = -8932$
10-20	$W(1) = 0.30$	$P(1) = -394$
20-30	$W(2) = 0.29$	$P(2) = -501$
30-40	$W(3) = 0.28$	$P(3) = -637$
40-50	$W(4) = 0.31$	$P(4) = -310$
50-60	$W(5) = 0.28$	$P(5) = -637$
60-70	$W(6) = 0.24$	$P(6) = -1665$
70-80	$W(7) = 0.24$	$P(7) = -1665$
80-90	$W(8) = 0.24$	$P(8) = -1665$
90-100	$W(9) = 0.23$	$P(9) = -2116$

Output parameters of the model are as follows: (a) Soil moisture contents of each layer ( $\text{cm}^3 \text{cm}^{-3}$ ), (b) Soil water potential of each layer (cm H<sub>2</sub>O) (c) Unsaturated hydraulic conductivity of each layer ( $\text{cm h}^{-1}$ ) and (d) Evaporation from soil surface ( $\text{cm h}^{-1}$ )

For testing the soil moisture dynamic in accordance with the results from the simulation model, field measurements at one-week intervals, at same time, the volumetric soil contents were measured from 3 parallels of each sample (moisture dynamics of the model are observed according to initial field value). Results obtained both from field and the model is interpreted in Figs. 1-3.

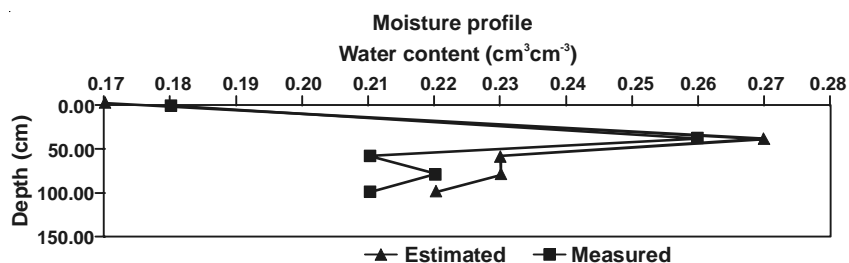


Fig. 1. Measured and estimated soil moisture dynamics (19 Oct. 1998)

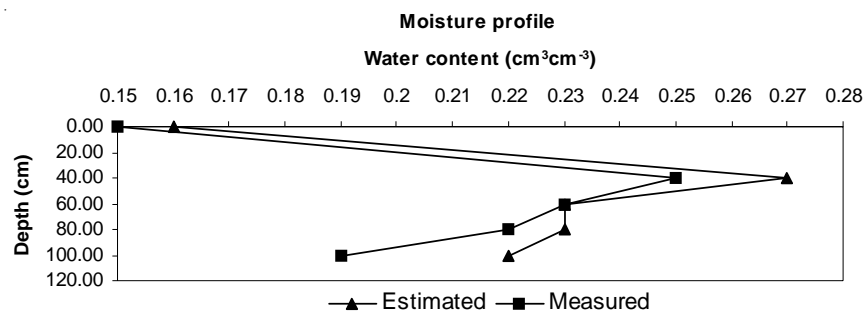


Fig. 2. Measured and estimated soil moisture dynamics (26 Oct. 1998)

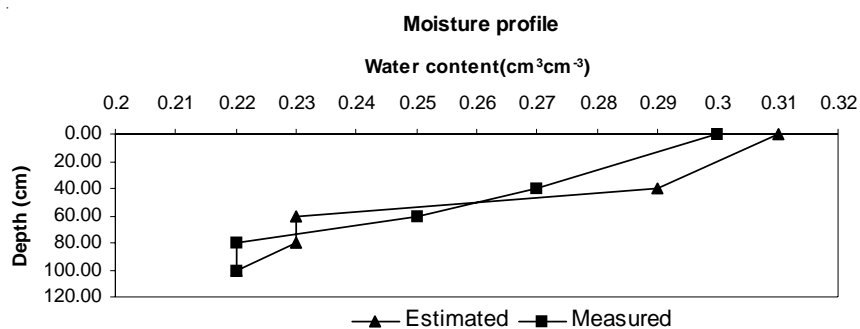


Fig. 3. Measured and estimated soil moisture dynamics (Rainfall = 1.8 cm) (02 Nov. 1998)

For the solution of soil moisture dynamic final differences method were used and stability state was observed in the solution. Present results revealed that the model and field measurements are generally in accordance. Further studies conducted by the authors with the object to integrate a model developed in this study into Energy-Mass Transfer and Plant Development Models in relation with other blocks, *i.e.* the soil on movement Blocks and Plant Development Blocks. Two approach proposed in the study could be used for practical studies of bare soils water dynamic. Water budget was simplified on soil surface (runoff, deep infiltration, plant uptake were not included). This model could be used in arid and semi arid regions particularly followed land where no runoff occurs.

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