

## **$\gamma$ -Ray Summing in Germanium Detectors and Its Effects on Nuclear Decay Parameters**

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The random, genuine and coincidence summing due to photons in germanium detectors are studied as a function of source detector distances. The radioactive isotope of  $^{133}\text{Ba}$  has been used for the purpose of genuine sum peaks. The source to detector distances kept at 2.5, 5.0 and 10.0 cm have revealed the exponential fall in sum peak areas. A typical random sum-peak at 867 keV due to 356 keV and 511 keV photo peaks from  $^{133}\text{Ba}$  and  $^{22}\text{Na}$  has been observed. The random summing has been found to increase with increase in the energies of the photo peaks being added. This phenomenon is about 2.4 % of the genuine (356 + 81) keV sum peak. The 81 keV photons having intensity as high as  $34.2 \pm 1.9$  units/100 decays of the parent nucleus could not show any summing with 511 keV peak. The resolving time of the detector found to depend strongly on count rate is determined as  $1.8 \pm 0.1$  microseconds.

**Key Words: Resolving time, Random and genuine sum-coincidences, Nuclear decay parameters, Conversion coefficients, Fluorescence yields, Relative K-capture probabilities.**

### **INTRODUCTION**

Germanium detectors have extensively been used for precise measurements of  $\gamma$ -ray intensities. Although, in performing  $\gamma$ -ray singles measurements with germanium detectors genuine summing of energies of  $\gamma$  radiations plays a vital role in intensity measurements of such radiations, the random summing however also plays an important role in determination of K-capture probabilities, K-shell fluorescence yields and absolute photo peak detection efficiencies of germanium detectors in sum-spectra. In very close geometries the summing of pulses due to cascading radiations from a radioactive source observed in a single detector has proved to be very useful for the determination of some nuclear decay parameters<sup>1-7</sup>. Detailed mechanism and effect of genuine summing in Ge-detector has been investigated by McCallum and Coote<sup>1</sup> and Gehrke *et al.*<sup>2</sup>. However the phenomenon of chance or random summing, which is also equally important when the distance between the

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source and detector is small, has not been probed thoroughly. Firstly, if two or more  $\gamma$ -rays are emitted in cascade, any two of them may deposit energy in the detector to form a composite pulse indistinguishable from that due to a single event. Other radiations, which could also be in true coincidence, include x-rays,  $\beta$ -particles and their associated bremsstrahlung and annihilation radiation from positron decay. If these radiations are not excluded from the detector by absorbers, their influence on the spectrum may be important, *i.e.*, a true count from full energy photo peak is lost. This results in an inaccurate determination of  $\gamma$ -ray intensities, while performing the measurements at very close geometries<sup>3</sup>. And secondly, while looking for genuine summing in detectors, chance or random summing also becomes very important thus influencing the conclusions drawn from the sum spectra<sup>8</sup>. Schmidt-Ott and Fink<sup>9</sup> used K-conversion electron-K x-ray coincidences to determine the K-capture probabilities and conversion coefficients. Mahapatra and Mukherjee<sup>10</sup> reported quantitative analysis of sum peaks for obtaining  $P_k$  values to the excited states of 383 and 437 keV levels. Single HpGe detector for  $\gamma$ - $\gamma$  sum peaks was used by Singh and Singh<sup>11</sup> to obtain sum peak intensities by placing the nuclear probe in various organic and inorganic environments. Correction equations to coincidence-summing effects were developed<sup>12</sup> for efficiency calibrations and proved worthwhile even with very short source to detector distances. The features of  $\gamma$ -ray spectra in the presence of significant contributions from x-rays to coincidence summing are reported<sup>13</sup> and compared with computed values in the decay of  $^{133}\text{Ba}$ . Novkovic *et al.*<sup>14</sup>, studied coincidence summing of x- and  $\gamma$ -rays in the decay of  $^{133}\text{Ba}$ . They determined peak count rates using the theory based on deposited energies in the detector<sup>14</sup> and thereafter probabilities for those events. Approach to K-electron-capture probabilities to the two excited states in the decay of  $^{133}\text{Ba}$  has also been exercised in some earlier works<sup>15</sup>. In the present measurements we have investigated the effects of genuine summing in  $^{133}\text{Ba}$  as a function of energy and source to detector distances and random summing with ( $^{133}\text{Ba} + ^{22}\text{Na}$ ) sources, extensively.

## THEORY

**Correction equations of coincidence summing and sum peak areas:** As the life times of the nuclear levels for  $\gamma$  decay are usually much shorter than the charge collection time in a semi-conductor detector, the two  $\gamma$ -rays emitted in cascade may happen to deposit energy in the detector to form a composite pulse indistinguishable from that due to the single event. The areas of peaks corresponding to these  $\gamma$ -rays will therefore get affected. Thus the intensity measurements will be affected by two kinds of summing effects, as sum peak due to accidental coincidence counts and secondly, the sum coincidence peak produced due to the cascading events detected in the detector. The accidental coincidences depend on the charge collection time and the count rate of the events being detected. This, in addition to high detection efficiency of the detector, puts an essential need to apply sum-coincidence corrections in order to evaluate precise  $\gamma$ -ray intensities. It has been found that none of the

previously published results for relative  $\gamma$ -ray intensities were reported to be corrected for such corrections. The three kinds of sum effects taken into account in the present work, (a) the sum of cascade  $\gamma$ -rays for themselves, (b) the sum of them for the crossover  $\gamma$ -rays and (c) the chance coincidence sum are explained below:

Consider a decay scheme, as shown in Fig. 1(a). If the activity of the source, emitting radiations is assumed as A, the  $\gamma$ -ray ( $\gamma_1$ ) emission probability as  $P_1$  and the full energy peak efficiency of the detector for  $\gamma_1$  as  $\epsilon_1$ , then the pulse rate  $N_1$ , resulting from the total absorption of  $\gamma$ -rays of energy  $E_1$  will be given as;

$$N_1 = AP_1\epsilon_1$$

As the emission of  $\gamma_1$  photon is followed by an emission of  $\gamma_2$  photon, the cascading radiations  $\gamma_1$  and  $\gamma_2$  or converted x-ray from internal conversion process, may enter the sensitive volume of the detector within its resolving time. Thus, the photon  $\gamma_1$  will be lost from the full energy peak. Then the rate of pulses,  $N_1'$  actually recorded under the photo peak of  $\gamma_1$  will be given as;

$$N_1' = AP_1\epsilon_1 - AP_1\epsilon_1\epsilon_2^T b_2 W_{12}(0^\circ) - AP_1\epsilon_1\epsilon_{kx}^T b_2 \alpha_k^2 \omega_k$$

where  $b_2$  = fraction of the decays from level that originates  $\gamma_1$  thus producing  $\gamma_2$  photons.  $\epsilon_2^T$  and  $\epsilon_{kx}^T$  = total efficiencies of the detector at the energy of  $\gamma_2$  and mean Kx-ray energy, respectively,  $\alpha_k^{(i)}$  = K-internal conversion coefficient for  $\gamma_i$ , while  $\omega_k$  = K-shell fluorescence yield of the daughter element.  $W_{ij}(0^\circ)$  = directional correlation function of  $\gamma$ -rays  $\gamma_i$  and  $\gamma_j$  averaged over the solid angle subtended by the detector.

Further, the observed photo peak area under the full energy peak of  $\gamma_2$  will be reduced due to its sum-coincidences with  $\gamma_1$ -photons or with Kx-rays produced due to internal conversion process which compete the  $\gamma_1$ -photon emission. The rate of pulses recorded under the  $\gamma_2$ -photo peak will therefore be given by;

$$N_2' = AP_2\epsilon_2 - AP_2\epsilon_2 f_1 \epsilon_1^T W_{12}(0^\circ) - AP_2\epsilon_2 \epsilon_{kx}^T f_1 \alpha_k^1 \omega_k$$

where various symbols mean similarly as discussed above.  $f_1$  = fraction of the events which populate the level that is further de-excited by  $\gamma_1$ -emission, thus producing  $\gamma_1$ -photons. In the Fig. 1(a), the full energy peak under  $\gamma_3$  will have additional photons contributed from the summing of  $\gamma_1$  and  $\gamma_2$  photons thus leading to a correction of negative type for the peak  $\gamma_3$ . In this case the observed area will include a term raised from the simultaneous depositions of  $\gamma_1$  and  $\gamma_2$  photons ( $\gamma_1 + \gamma_2 = \gamma_3$ , full energy peak) and the rate of pulses recorded would be given by;

$$N_3' = AP_3\epsilon_3 + AP_1\epsilon_1 b_2 \epsilon_2 W_{12}(0^\circ)$$

In case of decay schemes of the type, as in Fig. 1(b), the total events observed under the photo peak of  $\gamma_1$  will be given as;

$$N_1' = AP_1\epsilon_1 \{1 - b_2 \epsilon_2^T W_{12}(0^\circ) - \epsilon_{kx}^T b_2 \alpha_k^2 \omega_k - b_2^T b_3 \epsilon_3^T W_{13}(0^\circ) - b_2^T b_3 \alpha_k^3 \omega_k \epsilon_{kx}^T \}$$

The observed area under the  $\gamma_3$  peak will be;

$$N_3' = AP_3\epsilon_3 \{1 - f_2 \epsilon_2^T W_{23}(0^\circ) - \epsilon_{kx}^T f_2 \alpha_k^2 \omega_k - f_2^T f_3 \epsilon_3^T W_{13}(0^\circ) - f_2^T f_3 \alpha_k^3 \omega_k \epsilon_x^T \}$$

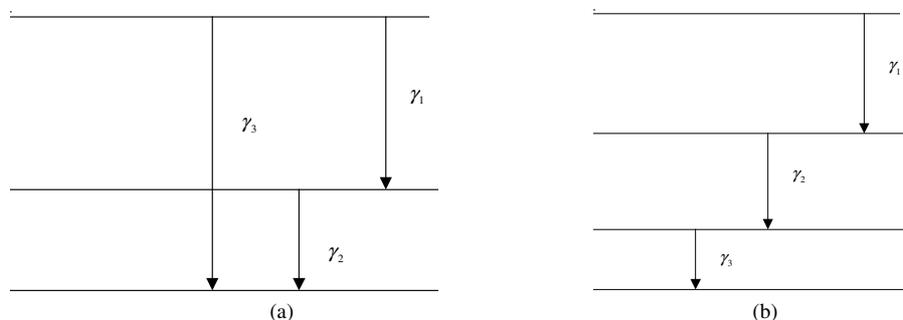


Fig. 1. (a-b): Simple decay schemes to derive expressions for coincidence summing corrections

where  $b_i^T = b_i(1 + \alpha_i^T)$  = fraction of the decays from the level depopulated by  $\gamma_i$ , thus resulting in the transition corresponding to  $\gamma_i$ .  $f_i^T(1 + \alpha_i^T)$  = fraction of the decays into the level populated by  $\gamma_i$ , which results from the transition corresponding to  $\gamma_i$ . The triple sum coincidence contribution will be negligibly small and it is not taken into account in the above equations.

The area under the sum peak ( $\gamma_1 + \gamma_2$ ) will be given by equation:

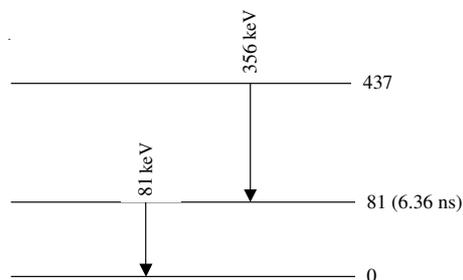
$$N(\gamma_1 + \gamma_2) = N_1 b_2^T b_3 \epsilon_3 W_{13}(0^\circ)$$

**Conformity of sum-coincidence corrections:** Sum-coincidence corrections are applied to 594 keV transition in the decay of  $^{147}\text{Nd}$ - $^{147}\text{Pm}$ . This photo peak of 594 keV transition energy experiences the contribution of additional events due to the summing of cascading transitions of (196 + 398) keV and (275 + 319) keV energies. The intensity measured in the present work is 0.586(14) which is almost half of the figure 0.95(6) from the table of isotopes<sup>16</sup>. To verify the accuracy of measurements the intensity balance was made to 686 keV level by using the relative  $\gamma$ -ray intensities measured in the present work. The branching to 686 keV level was obtained as 2.25 that agrees closely to the value of 2.2 adopted by the table of isotopes<sup>16</sup>.

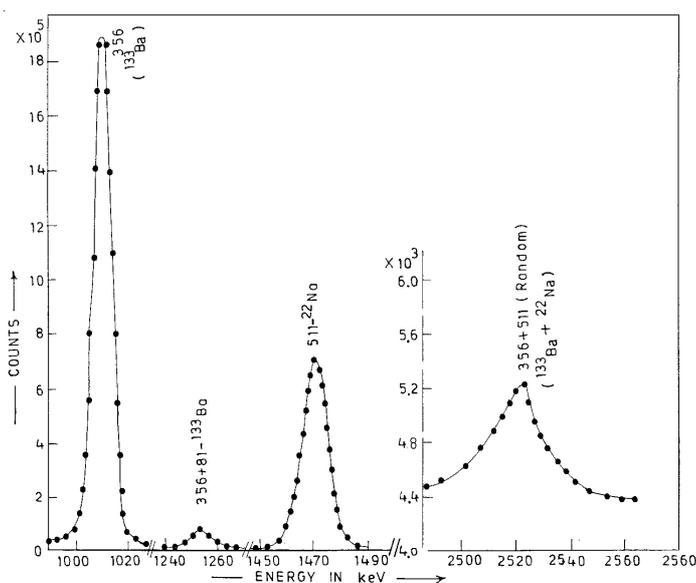
**Coincidence summing correction applied to 437 keV peak:** As shown in Fig. 2 the sum coincidence events due to 356 and 81 keV  $\gamma$ -rays will give rise to a sum peak at 437 keV energy. The spectrum was recorded with the source placed at a distance of 25 cm from the detector. The area under 437 keV sum peak has been evaluated as under:

$$N_{356+81}^{\text{sum}} = N_{437} = N_{356} \cdot \frac{1}{1 + \alpha_T^{81}} \epsilon_T^{81} \cdot W_{12}(0^\circ)$$

$N_{356}$  represents area under full energy peak at 356 keV.  $\epsilon_T^{81}$  is total efficiency at 81 keV,  $\alpha_T^{81}$  is total conversion coefficient at 81 keV  $\gamma$ -ray and  $W_{ij}(0^\circ)$  is the directional correlation function for the 356-81 keV cascade.

Fig. 2. 356-81 keV cascade in the decay of  $^{133}\text{Ba}$ 

The area under the 437 keV peak (Fig. 3) estimated from the above expression was obtained as 28976 (213), while the area observed under this peak was 28694(187). Because these areas are overlapping, it confirms that 437 keV is only due to the summing of 356 and 81 keV photons which are simultaneously detected by the detector due to its finite resolving time.

Fig. 3. Sum-peaks observed with  $^{133}\text{Ba}$  and ( $^{133}\text{Ba} + ^{22}\text{Na}$ ) sources

**Measurements:**  $\gamma$ -Ray spectra were recorded with 120 cc HPGe detector coupled to an ND-62 4K multi channel analyzer. The source to detector distances were kept at 2.5, 5.0 and 10.0 cm for different spectra. Genuine summing between  $81\gamma + \text{Kx-ray}$ ,  $276\gamma + \text{Kx-ray}$ ,  $356\gamma + \text{Kx-ray}$  and  $356\gamma + 81\gamma$  keV has been performed as a function of distance. The random summing between  $(356 + 511)$  keV  $\gamma$ -rays has been obtained using ( $^{133}\text{Ba} + ^{22}\text{Na}$ ) source. The sum coincidence corrections are applied to various  $\gamma$ -ray transitions recorded in large sized 120 cc HPGe detector.

**Determination of relative k-capture probabilities:** The relative K-capture probabilities to different excited states of  $^{133}\text{Cs}$  in the electron capture decay of  $^{133}\text{Ba}$  have been measured using kx-ray- $\gamma$ -ray sum coincidence summing in a single 7.4 cc intrinsic germanium detector. The equation for  $(356 + K_\alpha)$  sum peak is written as:

$$N_{356+K_\alpha}^{\text{sum}} = \omega_k \cdot \frac{I_{k_\alpha}}{I_{k_\alpha} + I_{k_\beta}} \cdot \epsilon_{k_\alpha} \left( P_K^{437} + \frac{\alpha_K^{81}}{1 + \alpha_T^{81}} \right) \cdot N_{356}$$

and the equation for  $(302 + K_\alpha)$  will be,

$$N_{302+K_\alpha}^{\text{sum}} = \omega_k \cdot \frac{I_{k_\alpha}}{I_{k_\alpha} + I_{k_\beta}} \cdot \epsilon_{k_\alpha} \cdot \left[ \left( 1 - \frac{I_{53}}{T_{383}} \right) \cdot P_K^{383} + \frac{I_{53}}{T_{383}} \left\{ P_K^{437} + \frac{\alpha_K^{53}}{1 + \alpha_T^{53}} \right\} + \frac{\alpha_K^{81}}{1 + \alpha_T^{81}} \right] \cdot N_{302}$$

where  $T_{383} = I_{383} + I_{302} + I_{223}$ .

Similar equations were written for other sum peaks. Then using the values of relative intensities of Kx rays and  $\gamma$ -rays measured in the present work and those of conversion coefficients and fluorescence yield from literature<sup>16</sup>, the equations were solved for  $P_K^{437}$  and  $P_K^{383}$ . The results are compared with earlier works<sup>9-10,17</sup> in Table-1. The results support the earlier findings of the authors<sup>18</sup> in the decay of  $^{147}\text{Nd}$ .

TABLE-1  
RELATIVE K-CAPTURE PROBABILITIES TO 437 AND 383 keV STATES OF  $^{133}\text{Cs}$

Energy level (keV)	K-Capture probabilities				Theoretical Present work
	Experimental				
	Present work	Mahapatra and Mukherjee <sup>10</sup>	Schmidtott and Fink <sup>9</sup>	Nicaise and Waltner <sup>17</sup>	
437	0.73(5)	0.76(6)	0.72(4)	0.747(96)	0.74
383	0.86(8)	0.87(14)	0.80(7)	–	0.83

## RESULTS AND DISCUSSION

The radioactive isotope of  $^{133}\text{Ba}$  has been used for the purpose of genuine sum peaks. The random summing between  $(356 + 511)$  keV  $\gamma$ -rays has been obtained using  $(^{133}\text{Ba} + ^{22}\text{Na})$  source. A typical random summing peak at 867 keV due to 511 and 356 keV photo peaks has been observed as shown in Fig. 3. The random summing has been found to increase with increase in the energies of the photo peaks being added. The summing in energies from  $^{133}\text{Ba} + ^{22}\text{Na}$  is observed only for  $(356 + 511)$  keV  $\gamma$ -rays. This phenomenon is about 2.4 % of the genuine  $(356 + 81)$  keV sum peak. The 81 keV photons having intensity as  $34.2 \pm 1.9$  units/100 decays of the parent could not show any summing with 511 keV peak. The random, genuine and coincidence summing due to photons in germanium detectors are studied as a function of source detector distances. As shown in Fig. 4, the source to detector distances kept at 2.5, 5.0 and 10.0 cm have revealed the exponential fall in sum

peak areas. The resolving time of the detector is found to depend strongly on the count rate. The resolving time of the detector in the present case is determined as  $1.8 \pm 0.1 \mu\text{s}$ .

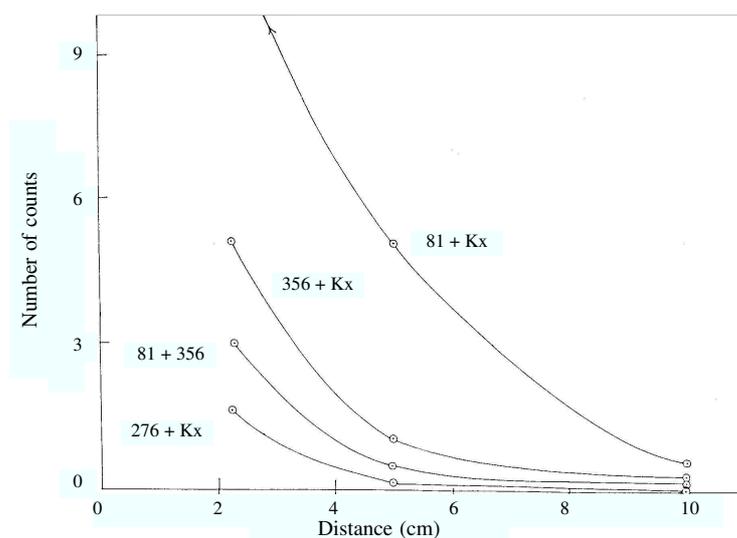


Fig. 4. Sum-peak areas as a function of source to detector distance

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