



General Dielectric Response and A.C. Conductivity†

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A simple non linear form of dielectric response function is proposed. The proposed dielectric response function is an essential extension of Debye linear response. The proposed dielectric response function is a simple one following L-C-R electric analogue of material to depict a non Debye response function. The function of dielectric response is capable to reproduce existing other response function along with some new features. The corresponding form of A.C. electrical conductivity and susceptibility are also extracted. The overall results obtained are good and encouraging.

Key Words: Electrical conductivity, Dielectric response.

INTRODUCTION

The advancement of material research on soft materials many complex materials have been encountered. The complex systems are like biopolymers, colloid systems, polymers, porous materials and many complex natural materials of other type. In most of the cases the mentioned material are non-crystalline in nature. The investigation of dielectric aspect of the materials demand more studies on structure and dynamics of the material. Dielectric spectroscopy has established its own advantages in this field compared with other techniques especially when one involve in investing electrical transport or characterization of bulk properties of the mentioned system. When an external field is applied on dielectric material, the electrical polarization of the material reaches to a steady value, after a period of time. Similarly when the applied electric field is withdrawn the polarization decay due to interaction with phonons and follows the identical law as the growth or decay function of dielectric polarization. The phenomenon is the basic relaxation process in a dielectric material. Therefore electrical response in many dielectric materials under external field is very similar to that of a series L-C-R circuit in general. From the analysis of impedance spectrum the role of mentioned phenomenological model had been established.

The electrical/optical conductivity of ion-conducting solid is mostly dependent on dielectric behaviour¹ of the material. The nature of frequency dispersion of electrical conductivity and or dielectric constant of the solid may be determined by

A.C. dielectric function or dielectric response. The simplest form of the linear response is the Debye response theory¹. Later many prominent phenomenological non-Debye responses²⁻⁶ have been proposed. In present work, authors make an attempt to develop a general response function following L-C-R model. The overall results obtained are concise and good.

THEORY

For the case of ideal dielectrics, Debye (1924) has formulated a relaxation model:

$$\epsilon^*(\omega) = \epsilon_{ca} + \frac{\epsilon_s - \epsilon_{ca}}{1 + i\omega\tau}$$

where, ϵ^* is the complex dielectric constant. ϵ_s and ϵ_{ca} are respectively, dielectric permittivity at the frequencies $\omega = 0$, which is static dielectric permittivity and $\omega = \infty$, which is optic dielectric permittivity. The single relaxation time τ corresponds to single relaxation process.

In general, however such relaxation are rarely observed in practice. Multiple relaxation or distribution of relaxation are found in real materials. The most commonly observed simple non Debye relaxation distributions are the Cole-Cole⁴, Davidson Cole⁵ or Havriliak Negami^{6,7} distribution.

According to our model, let us introduce a Differential equation:

$$\frac{d^2P}{dt^2} + \frac{A}{\tau} \frac{dP}{dt} + \frac{BP}{\tau^2} = \frac{\alpha E}{\tau^2} \quad (1)$$

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Solution of this differential equation:

$$P = \frac{\alpha E}{(B - \omega^2 \tau^2 + j\omega A \tau)} \quad (2)$$

Using the above equations one may express, the real and imaginary parts of the dielectric constant and hence the A.C. conductivity in terms of frequency and relaxation time.

$$\epsilon^* = \epsilon_{ca} + \frac{(\epsilon_s - \epsilon_{ca})}{(B - \omega^2 \tau^2 + j\omega A \tau)} \quad (3)$$

$$\epsilon' = \epsilon_{ca} + \frac{(\epsilon_s - \epsilon_{ca})(B - \omega^2 \tau^2)}{(B - \omega^2 \tau^2)^2 + (A\omega\tau)^2} \quad (4)$$

$$\epsilon'' = \frac{(\epsilon_s - \epsilon_{ca})A\omega\tau}{(B - \omega^2 \tau^2)^2 + (A\omega\tau)^2} \quad (5)$$

$$\sigma(\omega) = \sigma(0) + \frac{\epsilon_0(\epsilon_s - \epsilon_{ca})A\omega^2\tau}{(B - \omega^2 \tau^2)^2 + (A\omega\tau)^2} \quad (6)$$

For $\omega \ll \frac{1}{\tau}$

$$\sigma(\omega) = \sigma(0) + \frac{\epsilon_0(\epsilon_s - \epsilon_{ca})A\omega^2\tau}{B^2} \quad (7)$$

For $\omega \gg \frac{1}{\tau}$

$$\sigma(\omega) = \sigma(0) + \frac{\epsilon_0(\epsilon_s - \epsilon_{ca})}{\omega^2 \tau^3} \quad (8)$$

RESULTS AND DISCUSSION

The result obtained from this elementary is quite reasonable. The obtained expression for $\sigma(\omega)$ given in eqn. (6) has a singularity at $\omega\tau = \sqrt{B}$ and precisely at $\omega\tau = 1$ with $B^2 = 1 + A$.

In fact it limits, the choice of parameter A and B, moreover A and B may be determined theoretically with physical consistency condition. The overall nature of real ϵ may be compared with that of complex functional material.

$\omega \ll \frac{1}{\tau}$, $\omega^2 \tau^2$ may be neglected and reduces to Debye case. With increase in $\omega\tau$ so that $\omega^2 \tau^2$ has a contribution comparable to $\omega\tau$, gives a new result may explain many anomalous $\epsilon(\omega)$ or $\sigma(\omega)$ dispersion. For $\omega\tau = 1$ or $\omega\tau \gg 1$ the term $\omega\tau$ may be neglected.

Fig. 1 shows variation of $\sigma(\omega)$ with ω following eqn. (6). The eqn. (6) is the outcome of this present theoretical calculation. The low frequency part of A.C. conductivity $\sigma(\omega)$ shows about ω^2 variation and $\frac{1}{\omega^2}$ variation in the high frequency domain. In frequency domain $\omega\tau \approx 1$, $\sigma(\omega)$ has singularities,

representing physical phenomenon in charge transport mechanism in solid. The obtained response function in the frequency region $\omega\tau \gg 1$ corresponds to frequencies higher than phonon frequencies of system. The nature of $\sigma(\omega)$ in the mentioned region is the optical conductivity in a dielectric system. The proposed dielectric response and hence the calculated A.C. conductivity $\sigma(\omega)$ account the overall spectrum very well. The high frequency region provides a good qualitative account of optical conductivity showing Drude nature^{8,9}.

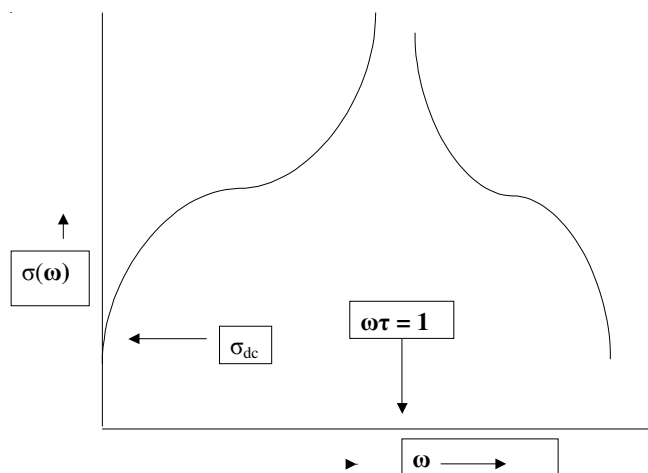


Fig.1. Variation of A.C. conductivity $\sigma(\omega)$ with frequency ω of electric field

Conclusion

The proposed dielectric constant provides a fair account of A.C. conductivity in both low and high frequency limit. In low frequency limit it can reproduce the pioneer Debye response however in the high frequency limit it provides more physical aspect of optical conductivity.

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